Angular Momentum

$\vec{L} = \vec{r} \times \vec{p}$

$L = mvr$ = Particle

Large Body

$v = \omega r$

$L = mvr = \boxed{m\omega r^2 = L}$

piece

Total Angular Momentum of Body =

$\sum_i m_i \omega r_i^2 = \left(\sum_i m_i r_i^2\right)\omega = I\omega$

I = moment of inertia $= \sum_i m_i r_i^2$

$\boxed{L = I\omega}$

Angular Impulse $= \vec{J}_\theta$

Constant Torque τ acting on body w/ I from time t_1 to $t_2 \Rightarrow \omega_1 \rightarrow \omega_2$

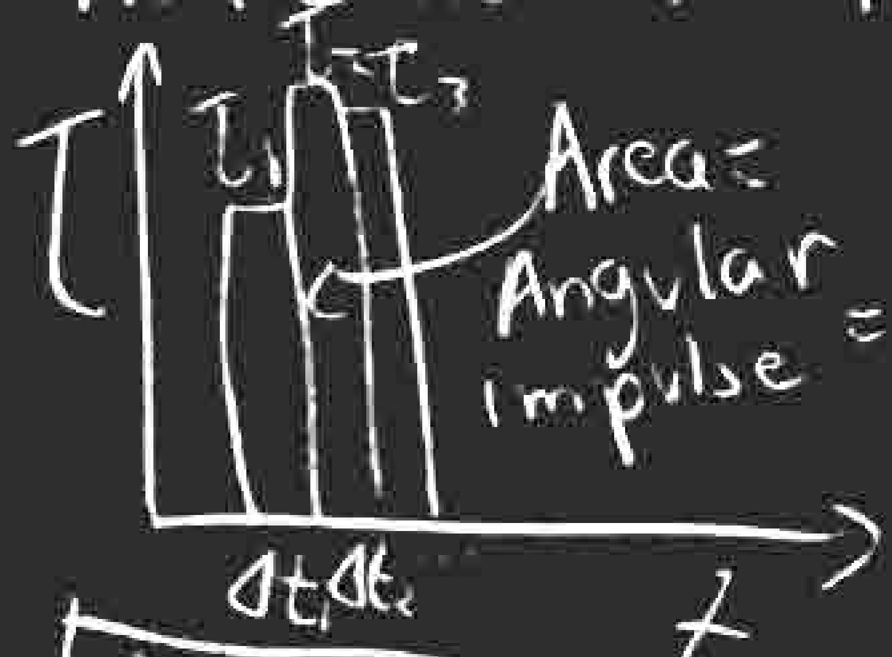
$\tau = I\alpha = I\left(\frac{\omega_2 - \omega_1}{t_2 - t_1}\right) \Rightarrow \tau(t_2 - t_1) = I\omega_2 - I\omega_1$

$= L_2 - L_1$

$$\boxed{\vec{J}_\theta = \vec{\tau}(t_2 - t_1) = \vec{L}_2 - \vec{L}_1}$$

$$\boxed{\vec{J}_\theta = \int_{t_1}^{t_2} \vec{\tau}\, dt = \Delta\vec{L} = \vec{L}_2 - \vec{L}_1}$$

non constant torque



Area = Angular impulse = Change in Angular Momentum

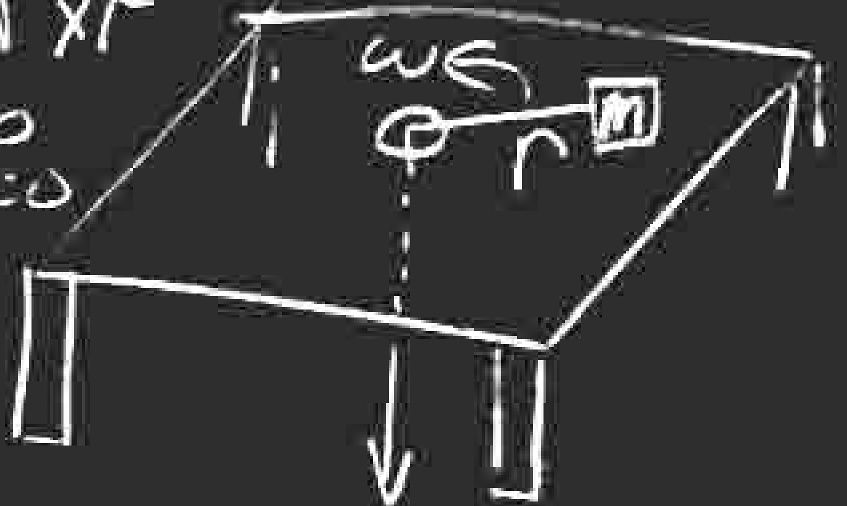
$$\boxed{\vec{J}_\theta = I\vec{\omega}_2 - I\vec{\omega}_1 = \vec{L}_2 - \vec{L}_1}$$

Conservation of Angular Momentum ⇒ resultant external torque on system = 0 ⇒ Angular Momentum is Constant

$\vec{\tau} = \vec{r} \times \vec{F}$

$r = 0$

$\Rightarrow \vec{\tau} = 0$



$m = .05\,kg$

$r_b = 0.2\,m \quad \omega_b = 3\,rad/s$

Pull cord ⇒ $r_a = 0.1\,m$ ⇒ $\omega_a = ?$

$L_{before} = L_{after}$

$m\omega_b r_b^2 = m\omega_a r_a^2$

$\omega_a = r_b^2\omega_o / r_a^2$

$= 3\,r/s \cdot (2m)^2/(1m)^2$

$= 12\,rad/s$

Δ Kinetic Energy $= K_a - K_b$

$K_b = \frac{1}{2} m r_b^2 \omega_b^2 = 9\times10^{-3}\,J$

$= \frac{1}{2}(.05\,kg)(2\,m)^2(3\,rad/s)^2$

$K_a = \frac{1}{2} m r_a^2 \omega_a^2 =$

$\frac{1}{2}(.05\,kg)(0.1\,m)^2(12\,rad/s)^2$

$= 3.6 \times 10^{-2}$ J

$\Delta K = K_a - K_b =$

3.6×10^{-2} J $- 9 \times 10^{-3}$ J

$= 2.7 \times 10^{-2}$ J increase

Pull on string for distance of 0.1m $\Rightarrow$ $F \cdot d = W = \Delta K$

$\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{L} = \vec{r} \times \vec{p}$

Right Hand Rule

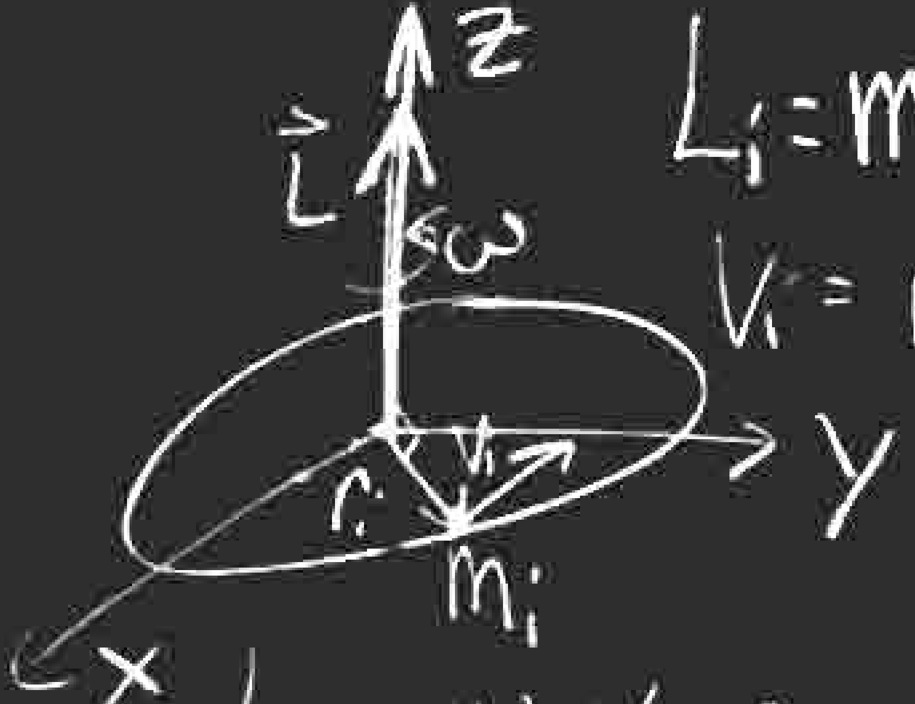
RH fingers follow rotation

Thumb = Vector Direction

$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\phi$

$|\vec{L}| = m|\vec{v}||\vec{r}| \cdot \sin\phi$



$L_i = m_i v_i r_i$

$v_i = r_i \omega$

$L_i = m_i v_i r_i$

$= m_i r_i \omega r_i = m_i r_i^2 \omega$

Total $\vec{L}$ is along z axis

$\vec{L} = L_z \hat{k}$, $L_z = \sum m_i r_i^2 \omega$

$L_z = (\sum m_i r_i^2)\omega$

$L_z = I\omega$

$\vec{L} = I\vec{\omega}$

Equilib^n

Conditions:

1. $\sum F_x = 0$, $\sum F_y = 0$
2. $\sum \vec{\tau} = 0$ around <u>any</u> axis.

Example. see-saw

P = pivot Force

$m_1 g = 400N$

$l_1 = 1.4m$, $l_2 = 1.6m$

$m_2 g = ?$

Conditions for Equil^n

(1) $\sum F_y = 0 = P - m_1 g - m_2 g$

(2) $\sum \tau = 0 = m_1 g l_1 - m_2 g l_2$

$\Leftrightarrow m_1 g l_1 = m_2 g l_2$

$m_2 g = m_1 g \frac{l_1}{l_2}$

$= 400N \frac{1.4m}{1.6m} = 350N$

(1) $\Rightarrow P - m_1 g - m_2 g = 0$

$P = m_1 g + m_2 g = 400N + 350N$

$= 750N$

Center of Gravity

body with a weight

act like weight concentrated

at a single point.

= Center of Gravity

Flat object, X-Y plane

made of small weights W_1, W_2, W_3

located at $(X_1, Y_1)(X_2, Y_2)\ldots$

$W = \Sigma W_i$

$\bar{X}$ = c.o.g

$= \quad \bar{X} = \dfrac{\Sigma W_i X_i}{\Sigma W_i}$