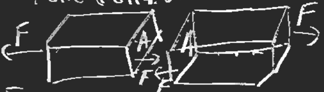
Elasticity:

Uniform Bar

Force distributed across A



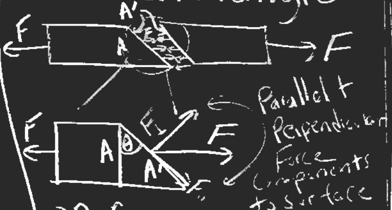
Each part of bar in equilibrium with other parts. ⇒ uniform Force across A everywhere

Define Stress $= \frac{F}{A}$

Bar is "under Tension"

⇒ Tensile Stress = Normal Stress b.c. perpendicular to area. Unit of Stress = Newtons/m² = Pascal. 1 Pa = 1 N/m²

Cut bar at an angle:



⇒ Define Tangential and Normal Stress:

Normal Stress $= F_\perp / A$

Tangential or Shear Stress $= F_\parallel / A$

Push on bar instead of Pull $F \rightarrow \square \leftarrow F$

Compressive stress

Shear Force = twist

Stress: Fluid under Pressure

Fluid at rest: shear force = 0

$\Rightarrow$ In Fluid $\Rightarrow$ all forces are perpendicular to surface of object in fluid.

Forces are uniform

$\Rightarrow \frac{\text{Force}}{\text{Area}} = \text{Pressure}$

Hydrostatic Pressure

$$P = \frac{F}{A} \text{ or } F = pA$$

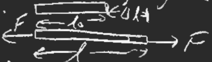
Solid in a fluid experiences hydrostatic pressure

Effect on object under stress = Strain

3 Types of strain corresponding to 3 types of stress.

Tensile, shear, hydrostatic Pressure

Bar under tension: orig length l_0 new length l.



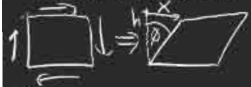
$\Delta l = l - l_0$

$$\text{Tensile Strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

Compressive Stress
length decreases

$$\text{Compressive Strain} = \frac{l_0 - l}{l_0} = \frac{\Delta l}{l_0}$$

Shear stress



$$\text{Shear strain} = \frac{x}{h} = \tan\phi \simeq \phi \text{ for small } \phi$$

generally true →

Hydrostatic Pressure
squashes object

$$\text{Volume Strain} = \frac{\Delta V}{V}$$

Behavior of material under stress is characterized by the material modulus
= ratio of stress to strain. ⇒ 3 moduli for 3 types of stress/strain

Longitudinal Modulus: Tensile or compressive stress/strain
(same for either comp or tensile)
= Young's Modulus

$$Y = \frac{\text{Tensile Stress}}{\text{Tensile strain}} = \frac{\text{Compressive Stress}}{\text{Compressive Strain}}$$

$$= \frac{F_\perp / A}{\Delta l / l_0} = \frac{l_0 F_\perp}{A \Delta l} \;:\; \text{Units} = N/m^2$$

Shear Modulus

$$\frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{h}{A}\cdot\frac{F_{\parallel}}{x} = \frac{F_{\parallel}/A}{\tan\phi} \approx \frac{F_{\parallel}/A}{\phi}$$

= Shear Modulus: Units: Force/area

Typically $\sim \frac{1}{2} - \frac{1}{3}$ of Young's Modulus

Hydrostatic Pressure: Bulk Modulus

$\Delta P \rightarrow -\Delta V/V$ Volume strain:

$B = \frac{-\Delta P}{\Delta V/V}$ → Pressure up ⇒ Volume down

Compressibility $k = \frac{1}{B} = \frac{-\Delta V/V}{\Delta P} = -\frac{1}{V}\frac{\Delta V}{\Delta P}$

Periodic Motion

Displace body from equilibrium with restoring force like Pendulum or spring + mass: $F = kx$

$F = -kx$

Stretch max distance $x = A$

Release and no friction ⇒ no energy loss ⇒ moves until reaches point where KE = 0, moves on

$E = U + K$, when $K = 0$
all energy = Potential
at location of max $X = A$ or $-A$
$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = U_0 = E$
= always total energy
Total range of motion
from $X = +A$ to $X = -A$
A = Amplitude of motion
Time for mass to return to
starting point = 1 Period

In 1 period, motion
undergoes 1 cycle
# cycles per unit
time = frequency $= \frac{1}{\text{Period}}$
f
1 cycle
per second = 1 Hertz
= 1 Hz = $1\,s^{-1}$
This type of motion w/o
energy loss = Simple
Harmonic Motion = SHM

acceleration is not
constant. $F = -kx = ma$
$\boxed{a = -\frac{k}{m}x}$
When X has max
Value of $A \Rightarrow$ accel
has max Value $a = -\frac{k}{m}A$
When $X = 0$, $a = 0$
but $V \neq 0$:

Energy Conservation
$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{Const.}$

When $V=0$ at $X_{max}=A$

$E=\frac{1}{2}kA^2=\text{Const}=$

$E=\frac{1}{2}kA^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2$

$kA^2=mv^2+kx^2$

$kA^2-kx^2=mv^2$

$V^2=\frac{k}{m}(A^2-X^2)$

$V=\pm\sqrt{\frac{k}{m}}\sqrt{A^2-X^2}$

If $V=0 \Rightarrow X=0$

$E=\frac{1}{2}mV_{max}^2=\frac{1}{2}kA^2$

$V_{max}^2=\frac{k}{m}A^2$ or $V_{max}=\pm\sqrt{\frac{k}{m}}A$

Periodic Motion is Projection of Rotational Motion on an axis.