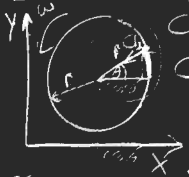
Periodic Motion

as a projection of Circular motion



$a_\perp = -\omega^2 r$

$a = -\frac{k}{m} X$

$\omega^2 = \frac{k}{m}$

Largest value of $X = r\cos\theta$

$X = A \Rightarrow X = A\cos\theta$

Let $\theta = \theta_0 = 0\ (t=0) \Rightarrow \boxed{\theta = \omega t}$

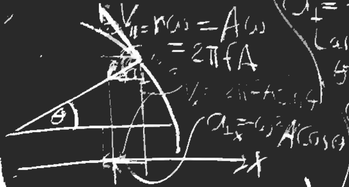
$\boxed{X = A\cos\omega t}$

1 Revolution in 2π radians

Rev/sec = frequency

$f = \omega / 2\pi$ or $\omega = 2\pi f$

$\boxed{X = A\cos 2\pi f t}$



Periodic Motion use V_x

$V = -2\pi f A\sin\theta = A\omega\sin\theta$

$\theta = \omega t \Rightarrow V = -\omega A\sin\omega t$

$\omega = 2\pi f \Rightarrow \boxed{V = -2\pi f A\sin 2\pi f t}$

Periodic Motion use $a_\perp$

$a_\perp = -\omega^2 r\cos\theta$

Largest $X = A \Rightarrow r = A$

$\theta = \omega t$ $\Downarrow$ Periodic

$a = -\omega^2 A\cos\omega t$

$\boxed{a = -4\pi^2 f^2 A\cos 2\pi f t}$

Let $\omega^2 = k/m \Rightarrow$

$a = -\omega^2 x$

$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{k}{m}} = f}$

freq. does not depend on amplitude A

Energy: $U = \frac{1}{2}kx^2$

$= \frac{1}{2}k(A\cos(\omega t+\theta_0))^2$

$U = \frac{1}{2}kA^2\cos^2(\omega t+\theta_0)$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m(-\omega A\sin(\omega t+\theta_0))^2$

$= \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\theta_0)$

$E = K + U =$

$\frac{1}{2}kA^2\cos^2(\omega t+\theta_0) + \frac{1}{2}kA^2\sin^2(\omega t+\theta_0)$

$= \frac{1}{2}kA^2(\cos^2(\omega t+\theta_0) + \sin^2(\omega t+\theta_0))$

$= \frac{1}{2}kA^2 \cdot 1$

$\boxed{E = \frac{1}{2}kA^2}$

Don't start at $\theta_0 = 0$ but at θ_0

$\theta = \theta_0 + \omega t$, $\omega = 2\pi/T = 2\pi f = \sqrt{\frac{k}{m}}$

$x = A\cos(\omega t+\theta_0)$, $v = -\omega A\sin(\omega t+\theta_0)$

$a = -\omega^2 A\cos(\omega t+\theta_0)$

$v = \pm\omega\sqrt{A^2-x^2}$, $a = -\omega^2 x$

$t=0$: $x_0 = A\cos\theta_0$, $v_0 = -\omega A\sin\theta_0$

$\omega^2 = k/m \Rightarrow$

$K = \frac{1}{2}m\frac{k}{m}A^2\sin^2(\omega t+\theta_0)$

$\boxed{K = \frac{1}{2}kA^2\sin^2(\omega t+\theta_0)}$

General Solution to Periodic Motion

$F = -kx$, $F = ma \Rightarrow ma = -kx$

$a = -\frac{k}{m} x$, $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} = -\frac{k}{m} x$, Let $\omega^2 = k/m$

$\frac{d^2x}{dt^2} = -\omega^2 x$. Solve with expression from previous board:

$x(t) = A\cos(\omega t + \theta_0)$

$\frac{dx}{dt} = A\frac{d}{dt}(\cos(\omega t + \theta_0)) = -\omega A \sin(\omega t + \theta_0)$

$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}\left(-\omega A \sin(\omega t + \theta_0)\right)$

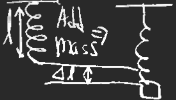
$= -\omega A \cdot \omega \cos(\omega t + \theta_0) = -\omega^2 A \cos(\omega t + \theta_0)$

$= -\omega^2 x$ ✓ $\Rightarrow$ Solves any eq. $\frac{d^2x}{dt^2} = -\omega^2 x$

$\Rightarrow$ when force acts linearly proportional to displacement $\Rightarrow$ Simple Harmonic Motion

IF NO ENERGY LOSS

Body Suspended from a Coil spring



weight $= mg$

Spring force $= k\Delta l$

Equilibrium: $k\Delta l = mg$

Spring displaced a distance X from new eq. position $l + \Delta l$

$F_{sp} = K(\Delta l - X)$ from spring

Total Force on body y

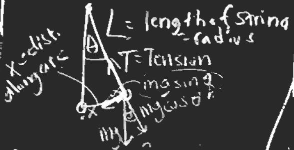
$F_{tot} = F_{sp} + F_g = K(\Delta l - X) - mg$

Since $K\Delta l = mg \Rightarrow$

$F_{tot} = mg - kx + mg = -kx$

regular spring $\omega = \sqrt{k/m}$

Simple Pendulum.



restoring force to bring back along x

$F = -mg\sin\theta$

Small angles: $\sin\theta \approx \theta$

$F = -mg\theta$ since mass goes along a circular path $x = L\theta \Rightarrow \theta = x/L$

$F = -mg\frac{x}{L} = -\left(\frac{mg}{L}\right)x$

$\Rightarrow$ of form $F = -kx$ where $k = \frac{mg}{L}$

$\Rightarrow$ SHM! $\Rightarrow$

$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}$

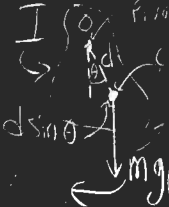
$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$

$\boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$

$\omega = 2\pi f \Rightarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$

Physical Pendulum

Rigid object pivoted around axis O, distance d from center of mass



$$\Sigma \tau = I\alpha$$

$$-mgd\sin\theta = I\frac{d^2\theta}{dt^2}$$

Small angles: $\sin\theta \approx \theta$

$$-mgd\theta = I\frac{d^2\theta}{dt^2} \Rightarrow$$ [central]

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta$$

defines ω

$$\boxed{\omega = \sqrt{\frac{mgd}{I}}}$$

$$T = \frac{2\pi}{\omega} \Rightarrow$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

Use SHM Eq'ns

Torsional Pendulum

Rigid body suspended by wire to fixed support $\Rightarrow$ twist by angle θ

$$\tau = -\kappa\theta$$

κ = torsion constant

angular form of Hooke's Law ($F = kx$)

$$\tau = -k\theta$$

$$I\alpha = -k\theta$$

$$I \frac{d^2\theta}{dt^2} = -k\theta$$

let ω be s.t.

$$\frac{d^2\theta}{dt^2} = -\frac{k}{I}\theta = -\omega^2\theta$$

$$\boxed{\omega = \sqrt{\frac{k}{I}}} \quad T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{I}{k}} = T}$$