April 23, 2003 Physics 202

EXAM 3

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitude. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
$k = 8.99 \times 10^9$ Nm^2 C^{-2}
$\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}
Speed of light, $c = 3.00 \times 10^8$ m/s.
Reference intensity for sound in decibels, $I_0 = 10^{-12}$ W/m^2.

PROBLEM 1

A 127 Ω resistor, a 0.141 μF capacitor and a 0.119 H inductor are connected in parallel to a voltage source with amplitude of 158 V

a. What is the resonance angular frequency? *(5 pts.)*

$$X_C = X_L = \frac{1}{\omega_0 C} = \omega_0 L \Rightarrow \omega_0 = 1/\sqrt{LC} = 1/\sqrt{(0.141\times 10^{-6}\text{F})(0.119\text{H})} = 7.72\times 10^{3}\ \text{rad/s}$$

$\boxed{7{,}720 \text{ rad/s}}$

b. What is the maximum current in the resistor at resonance? *(4 pts.)*

$I = V/R = 158\ \text{V}/127\ \Omega = 1.24\ \text{A}$

$\boxed{1.24 \text{ A}}$

c. What is the maximum current in the inductor at resonance? *(4 pts.)*

$$I_L = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{158\text{V}}{(7{,}720\ \text{rad/s})(119\ \text{H})} = 0.172\text{A}$$

$\boxed{0.172 \text{ A}}$

d. What is the maximum current in the branch containing the capacitor at resonance? *(4 pts.)*

$I_C = V/X_C = V\cdot\omega C = (158\ \text{V})(7{,}720\ \text{rad/s})(0.141\times10^{-6}\ \text{F}) = 0.172\ \text{A}$

$\boxed{0.172 \text{ A}}$

e. What is the maximum energy stored in the inductor at resonance? *(4 pts.)*

$$U_{max} = \tfrac{1}{2}LI_L^2 = \tfrac{1}{2}(0.119\ \text{H})(0.172\ \text{A})^2 = 1.76\times10^{-3}\ \text{J}$$

$\boxed{1.76\times10^{-3} \text{ J}}$

f. What is the maximum energy stored in the capacitor at resonance? *(4 pts.)*

$$U_{max} = \tfrac{1}{2}CV_C^2 = \tfrac{1}{2}(0.141\times10^{-6}\text{F})(158\ \text{V})^2 = 1.76\times10^{-3}\ \text{J}$$

$\boxed{1.76\times10^{-3} \text{ J}}$

PROBLEM 2

A transverse sine wave with an amplitude of 0.0835 m and a wavelength of 1.57 m travels from left to right along a horizontal string (along the *x* axis) of mass 1.26 g/m and tension 17.3 N. The origin of the string is the left end of the undisturbed string. At time $t = 0$, the left end of the string is at the origin and is moving downward.

a. What is the speed of the wave? *(5 pts.)*

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{17.3\text{N}}{1.26\times 10^{-3}\,\text{kg/m}}} = 117\,\text{m/s}$$

$$\boxed{117 \text{ m/s}}$$

b. What is the frequency of the wave in Hz? *(5 pts.)*

$v = f\lambda \Rightarrow f = v/\lambda = (117 \text{ m/s})/(1.57 \text{ m}) = 74.6 \text{ s}^{-1}$

$$\boxed{74.6 \text{ Hz}}$$

c. What is the maximum magnitude of the transverse (y) velocity? *(5 pts.)*

$v_y = A\omega = A\cdot 2\pi = (0.0835\text{m})(2\pi)(74.6 \text{ Hz}) = 39.1 \text{ m/s}$

$$\boxed{39.1 \text{ m/s}}$$

d. What is the transverse (y) position of the string at $x = 1.20$ m and $t = 0.0240$ sec? *(5 pts.)*

Wave travels to the right and at x=0, the string is going down: $y(x,t) = -A\sin(\omega t-kx)$

$k = 2\pi/\lambda = (2\pi \text{ rad})/(1.57 \text{ m}) = 4.00 \text{ rad/m}$

$y(1.20 \text{ m}, 2.62 \text{ s}) = -(0.0835 \text{ m})\sin(2\pi(74.6 \text{ Hz})(0.0240 \text{ s}) - (4.00 \text{ rad/m})(1.20\text{m})) =$

$-(0.0835 \text{ m})\sin(11.24 - 4.80) = -(0.0835 \text{ m})\sin(6.44) = 0.0136 \text{ m}$

$$\boxed{-14.0 \text{ cm}}$$

e. What is the transverse (y) velocity of the string at $x = 1.20$ m and $t = 0.0240$ sec? *(5 pts.)*

$v_y(x,t) = -A\omega\cos(\omega t-kx) = -(39.1 \text{ m/s})\cos(6.44) = -38.6 \text{ m/s}$

$$\boxed{-38.6 \text{ m/s}}$$

PROBLEM 3

A point source producing sound at 1.20 kHz with a power of 1.00 μW travels to the right at a speed of 98.0 ft/s relative to air. Ahead of it is a reflecting surface moving to the left at 216 ft/s relative to the air. The speed of sound in air is 1080 ft/s.

a. What is the sound level in decibels heard by an observer travelling 3.00 m away at the same speed as the source? *(5 pts)*

$$I = \frac{P}{4\pi r^2} = \frac{1.00\times10^{-6}\,\mathrm{W}}{4\pi(3.00\mathrm{m})^2} = 8.84\times10^{-9}\,\frac{\mathrm{W}}{\mathrm{m}^2} \Rightarrow \beta = 10\mathrm{dB}\log\left(\frac{I}{I_0}\right) = 10\mathrm{dB}\log\left(\frac{8.84\times10^{-9}\,\mathrm{W/m^2}}{10^{-12}\,\mathrm{W/m^2}}\right) = 39.5\mathrm{dB}$$

$\boxed{39.5 \text{ dB}}$

b. Find the wavelength in feet of the sound emitted toward the reflector by the source. *(5 pts.)*

$$\lambda = \frac{v}{f}\left(1-\frac{u}{v}\right) = \frac{1080\,\mathrm{ft/s}}{1200\mathrm{Hz}}\left(1-\frac{98.0\,\mathrm{ft/s}}{1080\,\mathrm{ft/s}}\right) = 0.818\mathrm{ft}$$

$\boxed{0.818 \text{ ft}}$

c. What is the number of wave-fronts arriving per second at the reflecting surface? *(5 pts.)*

$$f' = f\left(\frac{v+v_r}{v-v_s}\right) = 1200\mathrm{Hz}\left(\frac{1080\,\mathrm{ft/s}+216\,\mathrm{ft/s}}{1080\,\mathrm{ft/s}-98.0\,\mathrm{ft/s}}\right) = 1.58\times10^3\,\mathrm{Hz} = 1.58\mathrm{kHz}$$

$\boxed{1{,}580/\text{s}}$

d. What is the wavelength in feet of the reflected waves? *(5 pts.)*

$$\lambda = \frac{v}{f'}\left(1-\frac{v_r}{v}\right) = \frac{1080\,\mathrm{ft/s}}{1580\mathrm{Hz}}\left(1-\frac{216\,\mathrm{ft/s}}{1080\,\mathrm{ft/s}}\right) = 0.547\mathrm{ft}$$

$\boxed{0.547 \text{ ft}}$

e. What is the frequency of the reflected wave arriving back at the source? *(5 pts.)*

$$f_r = f'\left(\frac{v+v_s}{v-v_r}\right) = 1584\mathrm{Hz}\left(\frac{1080\,\mathrm{ft/s}+98.0\,\mathrm{ft/s}}{1080\,\mathrm{ft/s}-216\,\mathrm{ft/s}}\right) = 2.16\times10^3\,\mathrm{Hz} = 2.16\mathrm{kHz}$$

$\boxed{2160 \text{ Hz}}$

PROBLEM 4

A plane electromagnetic wave, with wavelength 3.0 m, travels in vacuum along the positive *x* direction with its electric field vector **E** of amplitude 300.0 V/m oriented along the positive y axis and then strikes a perfectly absorbing sheet of area 2.0 m^2.

a. What is the angular frequency of the wave? *(5 pts.)*

$\omega = 2\pi f = 2\pi c/\lambda = (2\pi)(3.00\times10^8 \text{ m/s})/(3.0 \text{ m}) = 6.8\times10^8$ rad/s

$$\boxed{6.8\times10^8 \text{ rad/s}}$$

b. What is the direction (+ or - and x, y, or z) and amplitude of the magnetic field? *(5 pts.)*

$$B_m = \frac{E_m}{c} = \frac{300.0\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = 1.00\times10^{-6}\,\text{T}$$

$$\boxed{1.0\times10^{-6}\,\text{T} \; +z}$$

c. What is the time averaged energy flow in W/m^2 associated with this wave? *(5 pts.)*

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300.0\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{H/m})(3.00\times10^8\,\text{m/s})} = 119\,\text{W/m}^2$$

$$\boxed{120 \text{ W/m}^2}$$

d. What is the rate at which momentum is delivered to the sheet? *(5 pts.)*

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\,\text{W/m}^2)(2.0\text{m}^2)}{3.00\times10^8\,\text{m/s}} = 8.0\times10^{-7}\,\text{N}$$

$$\boxed{8.0\times10^{-7}\,\text{N}}$$

e. What is the radiation pressure in Pascals exerted on the sheet? *(5 pts.)*

$$P = \frac{dp/dt}{A} = \frac{8.0\times10^{-7}\,\text{N}}{2.0\text{m}^2} = 4.0\times10^{-7}\,\text{Pa}$$

$$\boxed{4.0\times10^{-7}\,\text{Pa}}$$