May 12, 2003 Physics 202

FINAL EXAM

Print your name and section clearly on all nine pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8.5" x 11" sheet and no other references. The exam lasts exactly two hours.

(Do not write below)

SCORE:

SOLUTION KEY

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

Problem 5: __________

Problem 6: __________

Problem 7: __________

Problem 8: __________

TOTAL: __________

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$
$k = 8.99 \times 10^{9}\ N\ m^{-2}\ C^{-2}$
$e = 1.602 \times 10^{-19}\ C$
$m_e = 9.11 \times 10^{-31}\ kg$
$1\ eV = 1.602 \times 10^{-19}\ J$
$\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$
$1\ T = 1\ NA^{-1}m^{-1} = 10^4\ G$
$c = 2.998 \times 10^{8}\ m\ s^{-1}$

PROBLEM 1

The distance between the first and tenth minima of a double slit pattern is 18 mm on a screen 55 cm from the slits which are separated by 0.15 mm. Express wavelengths in nm.

a. What is the wavelength of the light used? *(8 pts.)*

$$\Delta y = \frac{D\lambda}{d}\Delta m = \frac{9D\lambda}{d} \Rightarrow \lambda = \frac{d\Delta y}{9D} = \frac{\left(0.15\times10^{-3}\,\mathrm{m}\right)\left(18\times10^{-3}\,\mathrm{m}\right)}{9\left(55\times10^{-2}\,\mathrm{m}\right)} = 545.5\mathrm{nm}$$

550 nm

b. If one of the slits mentioned above is covered with a thin piece of mica (n=1.58), the central point on the screen is then occupied by what had been the 7th-order bright fringe (m=7) before the mica was used. What is the thickness of the piece of mica? *(8 pts.)*

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x(n-1)}{\lambda} = 14\pi \Rightarrow x = \frac{7\lambda}{n-1} = \frac{7\left(545.5\times10^{-9}\,\mathrm{m}\right)}{1.58-1} = 6.58\times10^{-6}\mathrm{m}$$

6.6 μm

c. *(Unrelated to parts a and b above.)* A plane wave of monochromatic light is incident normally on a uniformly thick film of oil (n=1.30) that covers a glass plate (n=1.50). Destructive interference is observed for wavelengths of 500.0 nm and 700.0 nm. What is the thickness of the oil film? *(9 pts.)*

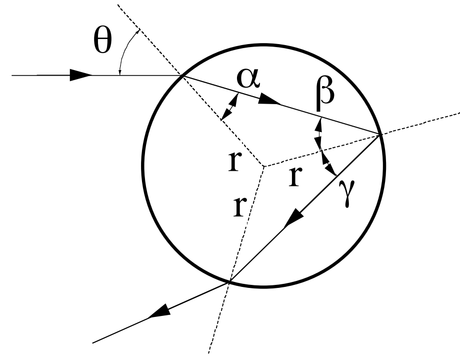
$$\frac{2Ln_0}{m+1/2} = \lambda_1, \frac{2Ln_0}{m+1+1/2} = \lambda_2, m = \frac{2Ln_0}{\lambda_1} - 1/2, 2Ln_0 = \lambda_2\left(\frac{2Ln_0}{\lambda_1} - 1/2 + 1 + 1/2\right)$$

$$\Rightarrow 2Ln_0\lambda_1 = 2Ln_0\lambda_2 + \lambda_1\lambda_2$$

$$\Rightarrow 2Ln_0\left(\lambda_1 - \lambda_2\right) = \lambda_1\lambda_2 \Rightarrow L = \frac{\lambda_1\lambda_2}{2n_0\left(\lambda_1-\lambda_2\right)} = \frac{(700\mathrm{nm})(500\mathrm{nm})}{2(1.30)(700\mathrm{nm}-500\mathrm{nm})} = 673\mathrm{nm}$$

670 nm

PROBLEM 2



A ray of red light is incident on a spherical drop of water of radius r. The angle of incidence is $\theta=45.2^0$. The index of refraction of red light in water is $n_w=1.330$, and in air is $n_a=1.000$.

a. What is the angle α? *(5 pts.)*

$$n_a \sin\theta = n_w \sin\alpha \qquad \sin\alpha = \frac{n_a}{n_w}\sin\theta = \frac{1.000}{1.330}\sin(45.2) = 0.533$$

$$\alpha = 32.2^\circ$$

32.2^o

b. Isosceles triangles have two equal angles, so β=α. What is the angle γ? *(5 pts.)*

$\theta_i = \theta_r$, so $\gamma = \beta = \alpha = 32.2^o$

32.2^o

c. For red light incident on a water-air interface from the water side, what is the maximum angle of incidence for which some light will still be transmitted to the air? *(5 pts.)*

$$\sin\theta_{max} = \frac{n_a}{n_w} = \frac{1.000}{1.330} = 0.752 \qquad \theta_{max} = 48.8^\circ$$

48.8^o

d. For red light incident on an air-water interface from the air side, what is the angle of incidence for which the reflected light is completely polarized? *(5 pts.)*

$\tan\theta_b=n_w/n_a=1.330/1.000 \qquad \theta_b = 53.1^o$

53.1^o

e. *(Unrelated to parts a—d above.)* The water drop has a net excess of 1500 electrons and is in an electric field of 120 V/m, directed downwards. What is the magnitude and direction of the electric force on the water drop? *(5 pts.)*

$$q = Ne = (1500)(-1.602 \times 10^{-19}C) = -2.4 \times 10^{-16}C$$

$$F = qE = (-2.4 \times 10^{-16}C)(-120\ V/m) = 2.9 \times 10^{-14}N$$

magnitude = 2.9×10^{-14} N, direction = upwards

2.9×10^{-14} N	upwards

PROBLEM 3

Consider a parallel-plate capacitor, with plate area 11.6 cm^2 and plate separation 0.254 cm, that has air between its plates. Ignore fringing fields.

a. What is the capacitance? *(5 pts.)*

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} C^2 N^{-1} m^{-2})(1.16 \times 10^{-3} m^2)}{2.54 \times 10^{-3} m} = 4.04 \times 10^{-12} \text{ F}$$

4.04×10^{-12} F

b. The capacitor is charged to 255 V. What is the magnitude of the electric field between the plates? *(5 pts.)*

$$E = \frac{V}{d} = \frac{255 \text{ V}}{2.54 \times 10^{-3} \text{ m}} = 1.00 \times 10^5 \text{ V/m}$$

1.00×10^5 V/m

c. What is the energy density in the region between the plates? *(5 pts.)*

$$E = \frac{1}{2}\varepsilon_0 E^2 = 0.5\,(8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2})(1.00 \times 10^5 \text{ V m}^{-1})^2 = 0.0446 \text{ J/m}^3$$

0.0446 J/m^3

d. The capacitor remains connected to the voltage supply as a slab of Teflon (κ=2.1), which completely fills the space between the plates, is inserted. What is the total energy stored in the capacitor? *(5 pts.)*

New capacitance = $C_{new} = \kappa C_{old} = 2.1\ (4.04 \times 10^{-12} \text{ F}) = 8.49 \times 10^{-12}$ F

$$E = \frac{1}{2} C_{new} V^2 = 0.5\,(8.49 \times 10^{-12} \text{ F})(255 \text{ V})^2 = 2.76 \times 10^{-7} \text{ J}$$

2.76×10^{-7} J

e. A 1.20×10^6 Ω resistor is connected across the plates of the capacitor (the Teflon slab remains in place). How much time elapses before half the charge on the capacitor is gone? *(5 pts.)*

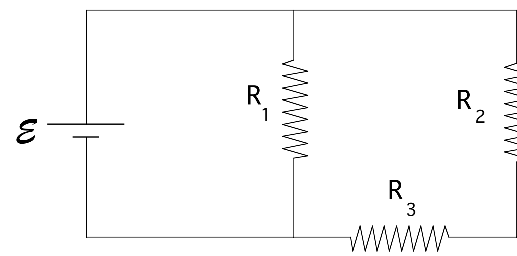
$$\tau = RC = (1.20 \times 10^6 \Omega)(8.49 \times 10^{-12} \text{C}) = 1.02 \times 10^{-5} \text{ s}$$

$$\frac{Q}{Q_0} = 0.5 = e^{-t/\tau} \quad t = -RC\ \ln(0.5) = 7.1 \times 10^{-6} \text{ s}$$

7.1×10^{-6} s

PROBLEM 4

A battery ($\mathcal{E}$=12.5 V) is connected to resistors R_1=49.9 Ω, R_2=28.7 Ω, R_3=60.4 Ω by copper wires of diameter 2.54 mm (you may neglect the resistance in the wires).



a. What is the current through R_2? *(5 pts.)*

$$I = \frac{\mathcal{E}}{R_2 + R_3} = \frac{12.5\ \text{V}}{28.7\ \Omega + 60.4\ \Omega} = 0.140\ \text{A}$$

0.140 A

b. What is the power (in Watts) dissipated in R_1? *(4 pts.)*

$$P = \frac{V^2}{R} = \frac{\mathcal{E}^2}{R_1} = \frac{(12.5\ \text{V})^2}{49.9\ \Omega} = 3.13\ \text{W}$$

3.13 W

c. How many electrons leave the negative terminal of the battery each second? *(7 pts.)*

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2 + R_3} = \frac{1}{49.9\ \Omega} + \frac{1}{28.7\ \Omega + 60.4\ \Omega} \rightarrow R_{eq} = 32.0\ \Omega$$

$$I_{tot} = \frac{\mathcal{E}}{R_{eq}} = \frac{12.5\ \text{V}}{32.0\ \Omega} = 0.391\ \text{A}$$

$$N = \frac{I}{e} = \frac{0.391\ \text{A}}{1.602\text{x}10^{-19}\ \text{C}} = 2.44\text{x}10^{18}\ /\text{s}$$

$2.44 \text{ x } 10^{18}$

d. What is the current density in the wire connected to the negative terminal of the battery? *(4 pts.)*

$$J = \frac{I}{A} = \frac{0.391\text{A}}{\pi\ (1.27\ \text{x}\ 10^{-3}\text{m})^2} = 7.71\ \text{x}\ 10^4\ \text{Am}^{-2}$$

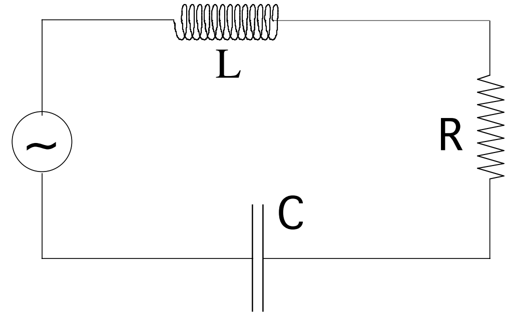
$7.71 \text{ x } 10^4$ A/m^2

e. What is the magnetic field caused by this current at a distance of 1.0 cm from the wire? *(5 pts.)*

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\ \text{x}\ 10^{-7}\ \text{N A}^{-2})(0.391\ \text{A})}{2\pi\ (0.01\ \text{m})} = 7.8\ \text{x}\ 10^{-6}\ \text{T}$$

$7.8 \text{ x } 10^{-6}$ T

PROBLEM 5



An inductor (L=10.5 mH), a resistor (R=271 Ω), and a capacitor (C=8.49 x 10^{-5} F) are connected in series to an AC voltage source of amplitude 165 V.

a. What is the resonance frequency of this circuit? *(5 pts.)*

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.05 \times 10^{-3}\text{H})(8.49 \times 10^{-5}\text{F})}} = 1.06 \times 10^3 \text{ s}^{-1}$$

1.06×10^3 s^{-1}

b. At resonance, what is the phase angle between the current and the voltage? *(5 pts.)*

At resonance, the current and the voltage are in phase.

0^o

c. At resonance, what is the maximum current in the inductor? *(5 pts.)*

$$I_{max} = \frac{V}{R} = \frac{165\text{ V}}{271\,\Omega} = 0.609\text{ A}$$

0.609 A

d. The inductor is a solenoid of length 8.4 cm and radius 0.33 cm, consisting of 150 turns of wire. At resonance, when the current is maximum, what is the magnitude of the magnetic field in the center of the solenoid? (Neglect the fringing field.) *(5 pts.)*

$$B = \frac{\mu_0 I N}{\ell} = \frac{(4\pi \times 10^{-7}\text{ NA}^{-2})(0.609\text{ A})(150)}{0.084\text{ m}} = 1.4 \times 10^{-3}\text{ T}$$

1.4×10^{-3} T

e. The frequency of the voltage source is now set to 0. After a long time, the voltage across the terminals of the voltage source is a constant 117 V. What is the charge on the capacitor? *(5 pts.)*

$Q = CV = (8.49 \times 10^{-5}\text{ F})\,(117\text{ V}) = 9.93 \times 10^{-3}$ C

9.93×10^{-3} C

PROBLEM 6

A diffraction grating has exactly 12,600 rulings uniformly spaced over a width of 25.400 mm. It is illuminated by yellow light from a sodium vapor lamp at wavelengths of 589.00 nm and 589.59 nm with normal incidence. Express wavelengths in nm.

a. At what angle (in degrees) does the first order maximum occur for λ = 589.00 nm? *(5 pts.)*

$$d = \frac{25.400\text{nm}}{12{,}600} = 2015.9\text{nm}, \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(1)(589.00\text{nm})}{2015.9\text{nm}}\right) = 16.9875°$$

17.0°

b. What is the highest order maximum that is observed for λ = 589.00 nm? *(5 pts.)*

can't see $m\lambda/d > 1 \Rightarrow m > d/\lambda = 2016\text{nm}/589.00\text{ nm} = 3.4 \Rightarrow$ can see 3rd order

3rd

c. What is the angular separation (in degrees) between the first order maxima of the two wavelengths (589.00 nm and 589.59 nm)? *(5 pts.)*

$$\theta' = \sin^{-1}\left(\frac{m\lambda'}{d}\right) = \sin^{-1}\left(\frac{(1)(589.59\text{nm})}{2015.9\text{nm}}\right) = 17.0059°$$

$$\Delta\theta = \theta' - \theta = 17.0059° - 16.9875° = 0.0184°$$

0.0184°

d. What is the smallest wavelength difference from 589 nm resolvable by this grating in the first order? *(5 pts.)*

$$R = Nm = (1.26 \times 10^4)(1) = 1.26 \times 10^4, \Delta\lambda = \frac{\lambda}{R} = \frac{589\text{nm}}{1.26 \times 10^4} = 0.0467\text{nm}$$

0.0467nm

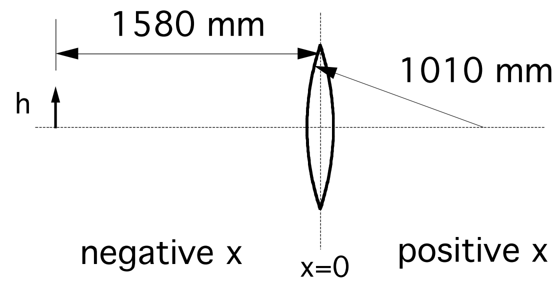
e. What is the smallest number of rulings that can resolve the two sodium lines (589.00 nm and 589.59 nm) in the first order? *(5 pts.)*

$$R = \frac{\lambda}{\Delta\lambda} = \frac{589\text{nm}}{0.59\text{nm}} = 998, n = \frac{R}{m} = \frac{998}{1} = 998 \text{ rulings}$$

998 Rulings

PROBLEM 7

A thin biconvex lens is made out of quartz (n=1.46) and symmetric about the vertical axis, with radii of curvature of both sides of 1010 mm. An object of height h=125 mm is placed 1580 mm away from the lens, as shown.



a. What is the focal length of the lens? *(5 pts.)*

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.46-1)\left(\frac{1}{1.010\text{ m}} - \frac{1}{-1.010\text{ m}}\right) = 0.911\text{ m}^{-1}$$

$$f = 1.10\text{ m}$$

1.10 m

b. What is the x coordinate of the image? *(5 pts.)*

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \rightarrow q = \frac{1}{1/f - 1/p} = \frac{1}{1/1.10\text{ m} - 1/1.58\text{ m}} = 3.60\text{ m}$$

q is positive, so the image is on the other side of the lens from the object

+3.60 m

c. What is the magnification of the image? *(5 pts.)*

$$M = \frac{-q}{p} = \frac{-3.60}{1.58} = -2.28$$

-2.28

d. The maximum thickness of the lens is 105 mm. How long does it take light to travel this distance through the lens? *(5 pts.)*

$$t = \frac{d}{c/n} = \frac{dn}{c} = \frac{(0.105\text{ m})(1.46)}{2.998 \times 10^8\text{ m/s}} = 5.11 \times 10^{-10}\text{ s}$$

5.11×10^{-10} s

e. The lens is replaced with a concave spherical mirror that has the same radius of curvature as either surface of the lens. Now what is the x coordinate of the image? (The mirror is facing the object) *(5 pts.)*

$$f = \frac{R}{2} = \frac{1.01\text{ m}}{2} = 0.505\text{ m}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \rightarrow q = \frac{1}{1/f - 1/p} = \frac{1}{1/0.505\text{ m} - 1/1.58\text{ m}} = 0.742\text{ m}$$

q is positive, so the image is on the same side of the mirror as the object

-0.742 m

PROBLEM 8

A plane electromagnetic wave propagating through empty space in the +x direction has an electric field vector described by **E**=(2.45 x 10^{-2} V/m) cos(kx-ωt) **j**. The frequency of the wave is f=8.77 x 10^{14} Hz.

a. What is the wavelength of the wave? *(5 pts.)*

$$\lambda = \frac{c}{f} = \frac{2.998 \text{ x } 10^{8} \text{m/s}}{8.77 \text{ x } 10^{14} \text{Hz}} = 3.42 \text{ x } 10^{-7} \text{m}$$

3.42 x 10^{-7} m

b. What is the average value of the Poynting vector for this wave? *(5 pts.)*

$$S_{av} = \frac{E^2}{2\mu_0 c} = \frac{(2.45 \text{ x } 10^{-2} \text{V/m})^2}{2(4\pi \text{ x } 10^{-7} \text{NA}^{-2})(2.998 \text{ x } 10^{8} \text{m/s})} = 7.97 \text{ x } 10^{-7} \text{ W m}^{-2}$$

7.97 x 10^{-7} W/m^2

c. The wave is normally incident on a perfectly reflecting mirror. What is the radiation pressure on the mirror? *(5 pts.)*

$$p = \frac{2S}{c} = \frac{2\,(7.97 \text{ x } 10^{-7} \text{W m}^{-2})}{2.998 \text{ x } 10^{8} \text{m s}^{-1}} = 5.31 \text{ x } 10^{-15} \text{ N/m}^2$$

5.31 x 10^{-15} N/m^2

d. The original wave and the reflected wave now form a standing wave. What is the distance between adjacent nodes of the standing wave? *(5 pts.)*

$$d = \frac{\lambda}{2} = \frac{3.42 \text{ x } 10^{-7} \text{m}}{2} = 1.71 \text{ x } 10^{-7} \text{m}$$

1.71 x 10^{-7} m

e. What is the amplitude of the electric field at an antinode? *(5 pts.)*

$$E = 2E_0 = 2\,(2.45 \text{ x } 10^{-2} \text{ V/m}) = 4.90 \text{ x } 10^{-2} \text{ V/m}$$

4.90 x 10^{-2} V/m