EXAM 2

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

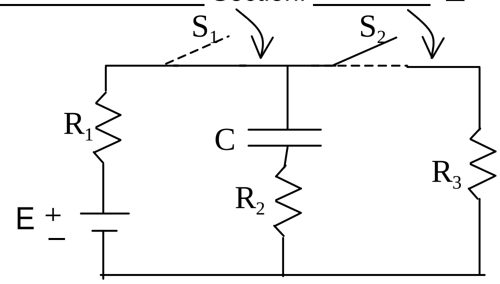
SOLUTION KEY

TOTAL: __________

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$
$k = 8.99 \times 10^{9}\ N\ m^2\ C^{-2}$
$e = 1.602 \times 10^{-19}\ C$
$\mu_0 = 4\pi \times 10^{-7} WbA^{-1}m^{-1}$
electron mass $m_e = 9.1 \times 10^{-31}$ kg
acceleration due to earth's gravity $g = 9.8\ ms^{-2}$

PROBLEM 1



In the circuit shown the resistors are R_1=53 Ω, R_2=77 Ω, R_3= 49 Ω. The capacitor, which starts out uncharged, has C = 23 μF, and the applied emf E = 4.5 V. At time t=0 the switch S_1, which has been open for a long time, is closed. (The switch S_2 stays open at this point.)

a. What is the voltage drop across resistor R_1 right after switch S_1 is closed? *(5 pts.)*

At t=0 the voltage drop across the capacitor is zero, so the current $I=E/(R_1+R_2)$. The voltage drop across R_1 is $IR_1 = ER_1/(R_1+R_2)$=(4.5 V)(53 Ω)/(53 Ω + 77 Ω) =

1.8 V

b. What is the voltage drop across the capacitor a long time after the switch S_1 has been closed, with S_2 left open? *(5 pts.)*

No current, and hence no voltage drop across resistors, so ΔV = E =

4.5 V

c. If the capacitor is uncharged at time t=0, at what time is the voltage drop across the capacitor half of the value in part (b)?

Voltage across capacitor is $V(t)=Q(t)/C$, and Q(t) satisfies $E = Q(t)/C+(R_1+R_2)dQ/dt$, so
$dQ/dt + Q(t)/((R_1+R_2)C)=E/(R_1+R_2) \Rightarrow Q(t)=(E/(R_1+R_2))(1-\exp(-t/(R_1+R_2)C))$.
So $Q(t)=E/(2(R_1+R_2))$ when $\exp(-t/(R_1+R_2)C)=1/2$, or $t=\ln 2\,(R_1+R_2)C$.
Thus, t= ln 2 (53 Ω+77 Ω)(23 μF) =

2.1×10^{-3} s

d. After the switch S_1 has been closed a long time, the switch S_1 is now reopened. (Switch S_2 is still open at this point.) How much energy is stored in the capacitor at this time? *(5 pts.)*

Energy $U = CV^2/2$ = (23 μF)(4.5 V)^2/2 =

2.3×10^{-4} J

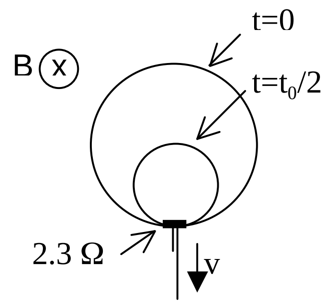
e. After a long additional time, switch S_2 is closed with switch S_1 kept open. How much power is dissipated in the resistor R_2 just after S_2 is closed? *(5 pts.)*

Current $I = V/(R_2+R_3)$, where V is the voltage across capacitor. Power $P = I^2R_2 = V^2R_2/(R_2+R_3)^2$ = (4.5 V)^2(77 Ω)/(77 Ω+49 Ω)^2 =

0.098 W

PROBLEM 2

A circular loop is in a magnetic field B=3.7 T directed into the page. At t=0 the loop radius $r=r_0=0.15$ m, and a force is applied to the loop so that the loop remains circular and its radius is time dependent: $r(t)=r_0-(r_0/t_0)t$ with $t_0 = 3.1$ s. The resistance of the loop, R, is 2.3 Ω.



a. Find the magnitude of the emf induced around the ring at time $t=t_0/2$. *(5 pts.)*

The magnitude of emf E is, by Faraday's law, dΦ/dt, where the magnetic flux Φ is $BA=B\pi r^2(t)$

$$= B\pi r_0^2\left(1-\frac{2t}{t_0}+\frac{t^2}{t_0^2}\right) \Rightarrow \frac{d\Phi_B}{dt} = B\pi r_0^2\left(-\frac{2}{t_0}+\frac{2t}{t_0^2}\right) = B\pi r_0^2\left(-\frac{2}{t_0}+\frac{1}{t_0}\right) = \frac{B\pi r_0^2}{t_0} = \frac{(3.7T)\pi(0.15m)^2}{3.1s} = 0.084V$$

$$\boxed{0.084 \text{ V}}$$

b. Find the magnitude of the current around the ring at time $t=t_0/2$. *(5 pts.)*

E = IR ⇒ I = E/R = (0.084 V)/(2.3 Ω) =

$$\boxed{0.036 \text{ A}}$$

c. Is the direction of the current flow in the loop at time $t=t_0/2$ clockwise or counterclockwise? *(5 pts.)*

Current acts to oppose change in flux, so current flows clockwise to create more magnetic field into the page.

$$\boxed{\text{Clockwise}}$$

d. What is the power dissipated in the loop at time $t=t_0/2$? *(5 pts.)*

Power = IE = (0.084 V)(0.036 A) =

$$\boxed{3.1 \times 10^{-3} \text{ W}}$$

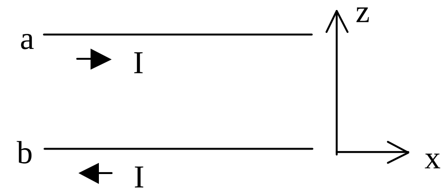
e. What is the magnitude of the force required to decrease the radius at the rate given at time $t=t_0/2$? *(5 pts.)*

Power = (force)(velocity); velocity=dr/dt = r_0/t_0, so force=(3.1×10^{-3} W)/(0.15m/3.1s)=

$$\boxed{6.4 \times 10^{-2} \text{ N}}$$

First Name: ____________________ Last Name:____________________ Section: ________ ■

PROBLEM 3



Two very long straight parallel wires carrying current I = 650 A in opposite directions are in the earth's gravitational field. The mass per unit length of the wires λ is 0.033 kg/m. The lower wire is supported and stationary at height z=0, and both wires point in the x-direction, with y=0.

a. What is magnitude of the magnetic field from wire b at wire a when wire a is a distance 0.015 m above wire b? *(5 pts.)*

Magnetic field a distance r from a wire carrying current I is

$$B(r) = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}TmA^{-1}\right)(650A)}{2\pi(0.015m)} = \frac{\left(2\times10^{-7}\right)(650)}{0.015}T =$$

$\boxed{8.7\times10^{-3}\ \mathrm{T}}$

b-c. There is a height z* such that when wire a is at z=z*, y=0 the net total force on wire a (gravitational plus magnetic) is zero (recall wire b is at y=z=0). Find the magnitude of z* and specify whether z*>0 (wire a above wire b) or z*<0 (wire a below wire b). *(10 pts.)*

Because currents are opposite, wires repel. So magnetic force on wire a is upward and can offset gravity when wire a is above wire b. Therefore, z*>0.

Magnitude: magnetic force per unit length between two wires separated by distance z* is

$\frac{F_B}{l} = \frac{\mu_0 I^2}{2\pi z^*}$, so force on unit length balances the gravitational force λg when

magnitude of z*	z*>0 or z*<0?
0.26 m	z*>0

d. For the situation in part (c), what is the magnitude of the total magnetic field at height z=z*/2? *(5 pts.)*

Field from wire a is equal and to the field from wire b, so the total magnetic field is twice:

$\boxed{2.0\mathrm{x}10^{-3}\ \mathrm{T}}$

e. For the situation in part (c), what is the magnitude of the total magnetic field at height z=z*/3? *(5 pts.)*

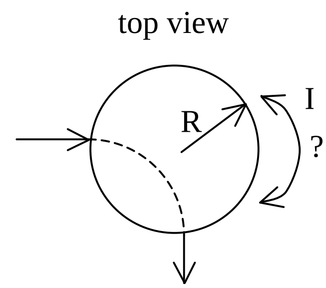
B=

$$\frac{\mu_0 I}{2\pi}\left(\frac{1}{z^*/3}+\frac{1}{2z^*/3}\right) = \frac{\mu_0 I}{2\pi z^*}(9/2) = \frac{9\left(2\times10^{-7}\,\mathrm{Tm/A}\right)(650\mathrm{A})}{2(0.26\mathrm{m})} = 2.3\times10^{-3}\,\mathrm{T}$$

$\boxed{2.3\times10^{-3}\ \mathrm{T}}$

First Name: ________________________ Last Name: _____________________ Section: ________ ■

PROBLEM 4



An empty solenoid has a current I=0.17 A flowing in the wire wound around it and has n=1600 turns/m. An electron (mass m = m_e = 9.1×10^{-31} kg, charge q = -e = -1.6×10^{-19} C) enters from the west with speed v=1.6×10^{7}m/s and leaves going south, as shown. The radius of curvature of the electron motion is equal to R, the solenoid radius. You may neglect fringing fields.

a. What is magnitude of the magnetic field in the solenoid? *(5 pts.)*

$B=\mu_0 nI=(4\pi\times10^{-7}\ \mathrm{TmA^{-1}})(1600\ \mathrm{m^{-1}})(0.17\ \mathrm{A})$ =

$\boxed{3.4\times10^{-4}\ \mathrm{T}}$

b. What is the magnitude of the magnetic force on the electron while it is inside the solenoid? *(5 pts.)*

Force = qvB = $(1.6\times10^{-19}\ \mathrm{C})(1.6\times10^{7}\mathrm{m/s})(3.4\times10^{-4}\ \mathrm{T})$ =

$\boxed{8.7\times10^{-16}\ \mathrm{N}}$

c. Is the current I around the solenoid flowing clockwise or counterclockwise? *(5 pts.)*

To get downward force on negatively charged particle coming from left, magnetic field must point into the page. To get magnetic field in this direction, current must be clockwise.

$\boxed{\text{clockwise}}$

d. What is the radius of curvature of the electron motion inside the solenoid? *(5 pts.)*

Particle in uniform magnetic field moves in circular path of radius
$mv/qB = (9.1\times10^{-31}\ \mathrm{kg})(1.6\times10^{7}\mathrm{m/s})/((1.6\times10^{-19}\ \mathrm{C})(3.4\times10^{-4}\ \mathrm{T}))$ =

$\boxed{0.27\ \mathrm{m}}$

e. If the current I is increased to 0.39 A and iron ($\mu=5000\mu_0$) inserted into the solenoid, what is then the magnitude of the magnetic field in the solenoid? *(5 pts.)*

$B=\mu nI=(5000)(4\pi\times10^{-7}\ \mathrm{TmA^{-1}})(1600\ \mathrm{m^{-1}})(0.39\ \mathrm{A})$ =

$\boxed{3.9\ \mathrm{T}}$