EXAM 3

Print your name and section <u>clearly</u> on all <u>five</u> pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

$$\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$$
$$k = 8.99 \times 10^9 Nm^{-2} C^{-2}$$
$$\mu_0 = 4\pi \times 10^{-7} WbA^{-1} m^{-1}$$

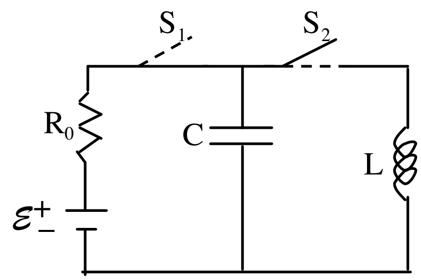
electron mass $m_e = 9.1 \times 10^{-31}$ kg

elementary charge $e = 1.6 \times 10^{-19}$ C

speed of light $c = 3.00 \times 10^8$ m/s

First Name: ____________________ Last Name: _____________________ Section: __________

PROBLEM 1



In the circuit shown the resistor has value $R_0 = 53\ \Omega$, the inductor L=16 μH, the capacitor C = 23 μF, and the applied DC voltage $\mathcal{E} = 4.5$ V. At time t=0 the switch S_2, which has been in the open position shown for a long time, is closed. (Switch S_1, which has been closed, stays closed at this point.)

a. What the current across resistor R_0 after the switch S_2 has been closed a long time? *(5 pts.)*

At long times there is no current across the capacitor and no voltage across the inductor. So $I=\mathcal{E}/R_0=(4.5\ \text{V})/(53\ \Omega)=$

$\boxed{0.085\ \text{A}}$

b. After the switch S_2 has been closed a long time τ, the switch S_1 is opened. How much energy is stored in the inductor at time τ? *(5 pts.)*

Energy = $LI^2/2 = (16\ \mu\text{H})(0.085\ \text{A})^2/2 =$

$\boxed{5.8 \times 10^{-8}\ \text{J}}$

c. What is the total electrical energy in the inductor and capacitor at time 2τ? *(5 pts.)*

Energy conserved in LC circuit, so is same as at time τ.

$\boxed{5.8 \times 10^{-8}\ \text{J}}$

d. What is the oscillation period of the circuit with S_1 open and S_2 closed? *(5 pts.)*

period $T = 2\pi/\omega$, and $\omega=1/(LC)^{1/2}$, so $T = 2\pi(LC)^{1/2} = (2\pi)(16\ \mu\text{H})^{1/2}(23\ \mu\text{F})^{1/2} =$

$\boxed{1.2 \times 10^{-4}\ \text{s}}$

e. Now S_1 is closed, S_2 is opened, and the DC voltage source replaced with an AC voltage source with maximum voltage 5.45 V at frequency 60.3 Hz. What is the time average of the power dissipated in the resistor after these changes are made? *(5 pts.)*

$I_{\max} = V_{\max}/X$ with $X = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$, and average power $P_{av}=I_{rms}^2R$

So

$$P_{av} = \frac{1}{2}\left(\frac{(5.45V)}{\sqrt{\left(\frac{1}{((60.3Hz)/(2\pi))(2.3\times 10^{-5}C)}\right)^2 + (53\Omega)^2}}\right)^2 (53\Omega) =$$

$\boxed{0.049\ \text{W}}$

PROBLEM 2

The wave function describing the vertical position y(x,t) for a traveling wave on a taut string is

$y(x,t) =$ (0.353 m) sin ($10.1\pi t + 2.59\pi x + \pi/4$) (t measured in seconds and x in meters).

a. What is the velocity at which the wave is traveling? *(5 pts.)*

For wavefunction proportional to $\sin(kx-\omega t+\phi)$, velocity = $d\omega/dk = (-10.1\pi\ s^{-1})/(2.59\pi\ m^{-1})$ =

$\boxed{-3.90 \text{ m/s}}$

b. What is the vertical position of an element of the string at t = 0, x = 0.124 m? *(5 pts.)*

$y = (0.353\ m) \sin((2.59\pi\ m^{-1})(0.124\ m) + \pi/4) = (0.353\ m)(0.975) =$

$\boxed{0.344 \text{ m}}$

c. What is the maximum magnitude of the transverse speed of the string? *(5 pts.)*

$$\frac{dy}{dt} = \frac{d}{dt}(A\sin(kx-\omega t+\phi)) = \omega A\cos(kx-\omega t+\phi)$$

Maximum value is $\omega A = (10.1\pi\ s^{-1})(0.353\ m)$ =

$\boxed{11.2 \text{ m/s}}$

d. The original wave combines with a second wave described by the wavefunction $y(x,t) =$ (0.353 m) sin ($11.2\pi t + 2.59\pi x + \pi/4$) (t measured in seconds and x in meters). What is the resulting beat frequency? (Don't worry about how it is possible for the string to support waves with two different velocities.) *(5 pts.)*

The beat frequency is $(11.2\pi/(2\pi))$ Hz – $(10.1\pi/(2\pi))$ Hz = 0.55 Hz

$\boxed{0.55 \text{ Hz}}$

e. For the combination of waves in part (d), what is the maximum displacement of the string at x=1.40 m that can occur at any time? *(5 pts.)*

For any x, the maximum displacement = 0.353 m + 0.353 m =

$\boxed{0.706 \text{ m}}$

PROBLEM 3

A siren speaker consists of a pipe closed on one end and open on the other. It emits sound at a fundamental frequency of 940 Hz. A steady wind is blowing with a speed of 20.0 m/s. Take the speed of sound in calm air to be 343 m/s.

a. What is the second-lowest resonant frequency (the first above the fundamental) that could be emitted by the siren? *(5 pts.)*

For pipe open at one end, second-lowest frequency is three times the fundamental frequency.
3×940 Hz =

$\boxed{2820 \text{ Hz}}$

b. Find the wavelength of the sound emitted at the fundamental frequency downwind of the siren (wind blowing in same direction as vector from siren to observer). *(5 pts.)*

Speed of sound downwind = 343 m/s + 20.0 m/s = 363 m/s.
Wavelength = (speed of sound)/(frequency) = (363 m/s)/(940 s^{-1}) =

$\boxed{0.386 \text{ m}}$

c. Firefighters are approaching the siren from various directions at 30.0 m/s. What frequency does a firefighter hear if he or she is approaching from a downwind position, so that he or she is moving in the opposite direction as the wind is blowing? *(5 pts.)*

frequency heard by firefighter $f' = (v + v_0)f/v$, where v is speed of sound and v_0 is speed of observer, so $f' = (363 \text{ m/s} + 30.0 \text{ m/s})(940 \text{ s}^{-1})/(363 \text{ m/s}) = 1018$ Hz

$\boxed{1020 \text{ Hz}}$

d. What frequency does a firefighter hear if he or she is approaching from an upwind position and moving with the wind? *(5 pts.)*

Speed of sound upwind = 343 m/s – 20.0 m/s = 323 m/s.

Frequency heard by firefighter $f' = (v + v_0)f/v = (323 \text{ m/s} + 30.0 \text{ m/s})(940 \text{ s}^{-1})/(323 \text{ m/s}) = 1027$ Hz

$\boxed{1030 \text{ Hz}}$

e. In the absence of wind, if the distance to the source is reduced by a factor of 3.1, what is the ratio of the intensity of the sound heard now to that before? *(5 pts.)*

Intensity falls off as square of the distance, so intensity increases by a factor of 3.1^2 =

$\boxed{9.6}$

PROBLEM 4

A linearly polarized electromagnetic wave of wavelength 1.43 cm is moving along the positive x axis. The electric field vector has a maximum value of 180 V/m and vibrates in the x-y plane. Assume the magnetic field component of the wave can be written in the form $B = B_{max}\sin(kx-\omega t)$.

a. Give the value of B_{max}. *(5 pts.)*

$B_{max} = E_{max}/c = (180\ \text{V/m})/(3.00 \times 10^8\ \text{m/s}) =$

$\boxed{6.00 \times 10^{-7}\ \text{T}}$

b. Find the frequency of this wave. *(5 pts.)*

frequency = speed of light/wavelength = $(3.00 \times 10^8\text{m/s}/(0.0143\ \text{m}) =$

$\boxed{2.10 \times 10^{10}\ \text{Hz}}$

c. Find the time average of the energy density of the wave. *(5 pts.)*

$$u_{av} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}(8.85 \times 10^{-12} C^2 N^{-1} m^{-2})(180V/m)^2 =$$

$\boxed{1.43 \times 10^{-7}\ \text{J/m}^3}$

d. Find the intensity of this wave. *(5 pts.)*

$$S_{av} = \frac{E^2}{2\mu_0 c} = \frac{(180V/m)^2}{(2)(4\pi \times 10^{-7} Tm/A)(3.00 \times 10^8 m/s)} =$$

$\boxed{43.0\ \text{W/m}^2}$

e. Find the radiation pressure this wave exerts at normal incidence on a perfectly reflecting sheet. *(5 pts.)*

$P = 2S/c = (2)(43.0\ \text{W/m}^2)/(3.00 \times 10^8\ \text{m/s}) =$

$\boxed{2.87 \times 10^{-7}\ \text{N/m}^2}$