February 16, 2005 Physics 202

EXAM 1

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

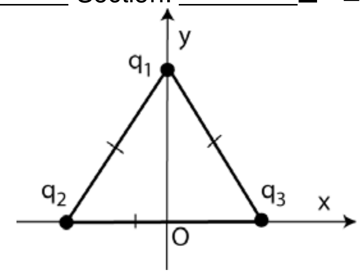
SOLUTION KEY

TOTAL: __________

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$
$k = 8.99 \times 10^{9}\ N\ m^2\ C^{-2}$
$e = 1.602 \times 10^{-19}\ C$

PROBLEM 1



Three charges $q_1 = +Q$, $q_2 = -Q$, and $q_3 = -Q$, where $Q = 1.35 \times 10^{-6}$ C are fixed in positions at the vertices of an equilateral triangle with each side 1.82 m long as shown.

a. What is the magnitude of the electric field at the origin (x=0, y=0)? *(5 pts.)*

At the origin the electric fields from q_2, and q_3 are equal in magnitude and opposite in direction so that only q_1 contributes:

$$E = k\frac{q_1}{r^2} = 8.99 \times 10^9 \,\mathrm{Nm^2C^{-2}} \frac{1.35 \times 10^{-6}\,\mathrm{C}}{(1.82 \sin 60^\circ)^2} =$$

4.89×10^3 N/C

b. What is the angle of the electric field at the origin with respect to the x-axis? *(5 pts.)*

Since only q_1 contributes, the field points away from q_1 or along the -y axis, which gives

-90°

c. What is the electric potential at the origin? *(5 pts.)*

$$V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + k\frac{q_3}{r_3} = kq_1\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3}\right) = 8.99 \times 10^9 \,\mathrm{Nm^2C^{-2}} \frac{1.35 \times 10^{-6}\,\mathrm{C}}{1.82\,\mathrm{m}}\left(\frac{1}{\sin 60^\circ} - \frac{2}{\cos 60^\circ}\right) =$$

-1.89×10^4 V

d. How much work in Joules is required to bring a fourth charge $q_4 = 1.20 \times 10^{-6}$ C from infinity to the origin? *(5 pts.)*

$W = q_4 V = (1.20 \times 10^{-6}\,\mathrm{C})(-1.89 \times 10^4\,\mathrm{V}) =$

-2.27×10^{-2} J

e. What is the magnitude of the net force exerted on q_4 at the origin by the charges q_1, q_2, and q_3 together? *(5 pts.)*

$F = q_4 E = (1.20 \times 10^{-6}\,\mathrm{C})(4.89 \times 10^3\,\mathrm{N/C}) =$

5.87×10^{-3} N

PROBLEM 2

The earth's electric field is measured at one point on its surface to be 89 N/C. If this were true everywhere over the earth's surface (treating it as a solid *conducting* sphere of radius 6400 km):

a. What is the charge inside and on the surface? *(8 pts.)*

All charge on surface so charge inside = 0 and charge on surface $Q = \varepsilon_0\Phi = \varepsilon_0 EA =$
$8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}\ (4\pi R^2)\ 89\ N/C = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2} (4\pi)(6.4\times10^6 m)^2 (89\ N/C) =$
4.05×10^5 C

Inside:	Surface:
0	4.1×10^5 C

b. What is the magnitude of the electric field inside the Earth? *(4 pts.)*

Electric field inside conductor = 0

0

c. What is the magnitude of the electric potential inside and on the surface of the Earth? *(8 pts.)*

Potential inside = surface = $kQ/R = 8.99 \times 10^9\ N\ m^2\ C^{-2}\ (4.05\times10^5\ C)/(6.4\times10^6 m) =$
5.69×10^8 V

Inside:	Surface:
570 MV	570 MV

d. What is the magnitude of the electric field outside the Earth at a distance of 6400 km *from the surface*? *(5 pts.)*

E is proportional to $1/R^2$, so E(2R) = (1/4)E(R) = (1/4) (89 N/C) =22.3 N/C

22 N/C

PROBLEM 3

A parallel plate capacitor has plate area of 620 cm^2, plate separation of 4.0 mm and is connected to a power supply that charges it to 130 V. It is then disconnected from the power supply.

a. Find the electric field between the plates *(4 pts.)*

$E_0 = V/d = 130V/4.0\times10^{-3}$ m $= 3.25\times10^4 V/m$

$3.3\times10^4 V/m$

b. Find the surface charge density on one of the plates *(4 pts.)*

$\sigma = Q/A = \varepsilon_0 E_0 = (8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2})(3.25\times10^4 V/m) = 2.88\times10^{-7}\ C/m^2 =$

$2.9\times10^{-7} C/m^2$

c. A sheet of insulating dielectric ($\kappa = 4.2$) is inserted that completely fills the space between the plates. Find the new electric field between the plates *(4 pts.)*

$E = E_0/\kappa = 3.25\times10^4 V/m/4.2 =$

$7.7\times10^3 V/m$

d. Find the capacitance of the capacitor with the dielectric *(4 pts.)*

$C = \kappa\varepsilon_0 A/d = (4.2)\ (8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2})(0.062\ m^2)/4.0\times10^{-3}$ m $= 5.76\times10^{-10}$ F or

$C = Q/V = \kappa A\sigma/V = 4.2(0.062\ m^2)(2.88\times10^{-7}\ C/m^2)/(130\ V) = 5.77\times10^{-10}$ F

0.58 nF

e. Find the induced surface charge density on each face of the dielectric *(4 pts.)*

$\sigma_{ind} = \sigma(\kappa - 1)/\kappa = 2.88\times10^{-7}\ C/m^2(4.2 - 1)/4.2 =$

$2.2\times10^{-7}\ C/m^2$

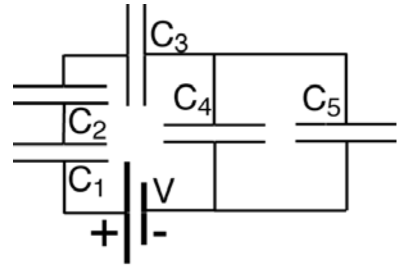
f. Find the magnitude and sign of the change in stored energy caused by inserting the dielectric *(5 pts.)*

$\Delta U = (-1/2)Q_0V_0(1 - 1/\kappa) = (-0.5)A\sigma V_0(1 - 1/\kappa) =$

$(-0.5)(0.062\ m^2)(2.88\times10^{-7}\ C/m^2)(130\ V)(1 - (1/4.2)) =$

-8.8×10^{-7} J

PROBLEM 4



A 7.0 V battery and five capacitors of capacitance each 5.0 μF form the circuit shown

a. What is the combined capacitance of C_4 and C_5? *(5 pts.)*

$C_{45} = C_4 + C_5 = 1.0 \times 10^{-5}$ F

10. μF

b. What is the total capacitance of the entire circuit? *(5 pts.)*

$C_{tot}^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + C_{45}^{-1} = 3(5.0 \times 10^{-6}\ \text{F})^{-1} + (1.0 \times 10^{-5}\ \text{F})^{-1} \Rightarrow C_{tot} = 1.43 \times 10^{-6}$ F

1.4 μF

c. What is the total charge on capacitor C_1? *(5 pts.)*

Since each capacitor has the same charge, $Q_1 = C_{tot}V = (1.43 \times 10^{-6}\ \text{F})(7.0\text{V}) =$

1.0×10^{-5} C

d. What is the total energy stored in this circuit? *(5 pts.)*

$U_0 = (1/2)C_{tot}V^2 = (0.5)(1.43 \times 10^{-6}\ \text{F})(7.0\text{V}) =$

3.5×10^{-5} J

e. After the whole circuit is immersed in oil (κ = 2.5), what is the total energy? *(5 pts.)*

$U = (1/2)\kappa C_{tot}V^2 = (0.5)(2.5)(1.43 \times 10^{-6}\ \text{F})(7.0\text{V}) =$

8.8×10^{-5} J