EXAM 3

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

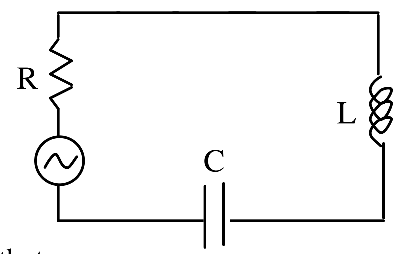
$$\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$$
$$k = 8.99 \times 10^{9} N m^{-2} C^{-2}$$
$$\mu_0 = 4\pi \times 10^{-7} T A^{-1} m^{-1}$$

electron mass $m_e = 9.11 \times 10^{-31}$ kg

elementary charge $e = 1.60 \times 10^{-19}$ C

speed of light $c = 3.00 \times 10^{8}$ m/s

PROBLEM 1



You have an AC generator (V_{max}=120.0V, f=60.0 Hz) that you would like to use to power a series RLC circuit in which the current in the capacitor leads the EMF of the generator by 30.0°. You already have a resistor (R=803 Ω) and a capacitor (C=4.00 μF), and add an inductor as shown.

a) What value of the inductance L yields a current in the capacitor that leads the EMF of the generator by 30.0°? *(5 pts)*

Want phase angle φ to be -30°. Use $\tan\phi=((X_L-X_C)/R)$, with $X_L=\omega L$ and $X_C=1/\omega C$ and solve for L:

$$L=\frac{1}{\omega}\left(R\tan\phi+\frac{1}{\omega C}\right)=\frac{1}{(2\pi)(60.0)}\left((803)\left(\frac{-1}{\sqrt{3}}\right)+\frac{1}{(2\pi)(60.0)\left(4\times10^{-6}\right)}\right)=$$

0.529 H

b) What is the maximum current flowing at any time in this circuit? *(5 pts)*

$$I_{max}=\frac{\Delta V_{max}}{\sqrt{R^2+(X_L-X_C)^2}}=\frac{\Delta V_{max}}{R\sqrt{1+\tan^2\phi}}=\frac{120V}{803\Omega\sqrt{1+1/3}}=$$

0.129 A

c) What is the time average of the power dissipated in the circuit? *(5 pts)*

$P_{av}=I_{max}^2R/2=(0.129\text{ A})^2(803\ \Omega)/2=$

6.68 W

d) What is the resonant angular frequency ω_0 of this circuit? *(5 pts)*

L=0.529H and C=4.00 μF; resonant angular frequency $\omega_0=1/(LC)^{1/2}=1/(0.529\times4.00\times10^{-6})^{1/2}=$

687 s^{-1}

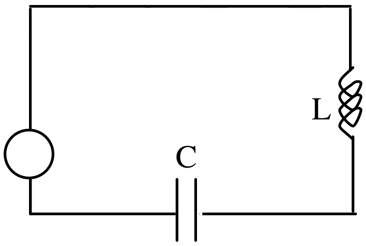
e) A new AC generator (with the same V_{max}) operating at the circuit resonant angular frequency, ω_0, is substituted for the original AC generator. What is the time average of the power dissipated in the circuit now? *(5 pts)*

At resonance, $P_{av}=V_{max}^2/2R=(120V)^2/(2\times803\Omega)=$

8.97 W

PROBLEM 2

An inductor having inductance L = 127 mH and a capacitor having capacitance C = 1.09 μF are connected in series with a current source. The circuit starts with the capacitor uncharged and the current source off at time t=0, at which time the current source is switched on and increases the current with time as I=at, where a=35.5 A/s is a constant.



a. Find the voltage across the inductor at time t=0.00250 s. *(5 pts.)*

$$V = L\frac{dI}{dt} = (0.127H)(35.5A/s) =$$

4.51 V

b. Find the voltage across the capacitor at time t=0.00250 s. *(5 pts.)*

charge on capacitor $Q=at^2/2$ and $V=Q/C$, so $V = \frac{at^2}{2C} = \frac{(35.5A/s)(0.0025s)^2}{(2)(1.09\times10^{-6}F)} =$

102 V

c. Find the energy in the capacitor at time t=0.00250 s. *(5 pts.)*

Energy = $CV^2/2 = (1.09\times10^{-6}\text{F})(102\text{V})^2/2 =$

0.00567 J

d. Find the energy in the inductor at time t=0.00250 s. *(5 pts.)*

Energy = $LI^2/2 = L(at)^2/2 = (0.127)((35.5)(0.00250))^2/2 =$

5.00×10^{-4} J

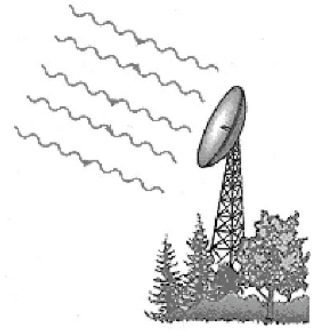
e. At time t=0.00250 s the current source is replaced with a wire without changing the current at that instant Find the total energy (the sum of the energy in the inductor and the capacitor) at time t=5.55 s. *(5 pts.)*

Total energy is the same for all t>2.50 s, so total energy = 0.00567 J + 0.00050 J =

0.00617 J

PROBLEM 3

A dish antenna having a diameter of 19.8 m receives (at normal incidence) a radio signal from a distant source, as shown. The radio signal is a continuous sinusoidal wave with frequency 121 MHz and electric field amplitude at the antenna surface $E_{max} = 0.208\ \mu V/m$. Assume the antenna absorbs all the radiation that falls on the dish.



a. What is the amplitude of the magnetic field in this wave? *(5 pts.)*

$B_{max} = E_{max}/c = (2.08\times10^{-7}V/m)/(3.00\times10^{8}m/s) =$

6.93×10^{-16} T

b. What is the intensity of the radiation received by this antenna? *(5 pts.)*

$I=E_{max}{}^{2}/(2\mu_0 c) = (2.08\times10^{-7}V/m)^2/((2)(4\pi\times10^{-7}TA^{-1}m^{-1})(3.00\times10^{8}m/s)) =$

5.74×10^{-17} Wm^{-2}

c. What is the total power received by the antenna? *(5 pts.)*

Power $P = IA = (5.74\times10^{-17}T/m^2)(\pi)(19.8/2)^2 =$

1.77×10^{-14} W

d. What is the total force exerted by the radio waves on the antenna? *(5 pts.)*

Force $F = P/c = (1.77\times10^{-14}\ W)/(3.00\times10^{8}\ m/s) =$

5.89×10^{-23} N

e. What is the wavelength of the radio waves? *(5 pts.)*

Wavelength $\lambda = c/f = (3.00\times10^{8}ms^{-1})/(1.21\times10^{8}s^{-1}) =$

2.48 m

PROBLEM 4

A transverse sinusoidal wave in the x-y plane with a period T = 25.3 ms travels in the negative x-direction along a string oriented along the x-axis. The mass per unit length of the string is μ=0.533 kg/m and the tension in the string is $\mathcal{T}$=489.3 N. At t=0, a particle on the string at x=0 has a transverse position of 2.01 cm above the x-axis, and has a transverse upward speed of 1.95 m/s away from the x axis. The transverse position of the wave can be written y=Asin(kx+ωt+φ).

a. What is the angular frequency ω of the wave? *(5 pts.)*

ω=2π/T = (2π)/(0.0253 s) =

$248\ s^{-1}$

b. What is the wavevector k of the wave? *(5 pts.)*

velocity $v=(\mathcal{T}/\mu)^{1/2} = (489.3N/0.533kg/m)^{1/2} = 30.3\ ms^{-1}$
$v=\omega/k \Rightarrow k=\omega/v = (248\ s^{-1})/(30.3\ ms^{-1}) =$

$8.20\ m^{-1}$

c. What is the phase angle φ of the wave? *(5 pts.)*

transverse position at t=0 is y=Asin(kx+ωt+φ)=Asinφ, and
transverse velocity = ωAcos(kx+ωt+φ)=ωAcosφ. Dividing:

$$\tan\phi = \frac{\omega y(t=0)}{dy/dt\,|_{t=0}} = \frac{(248s^{-1})(0.0201m)}{(1.95m/s)} = 2.56, \text{ and } \phi = \tan^{-1}(2.56) =$$

1.20 rad

d. What is the maximum amplitude of the transverse displacement? *(5 pts.)*

y(x=0,t=0)=Asinφ, so A=y(x=0,t=0)/sinφ = (0.0201m)/sin(1.20 rad) =

0.0216 m

e. What is the maximum transverse speed of the string? *(5 pts.)*

Maximum transverse speed = ωA = $(248\ s^{-1})(0.0216\ m)$ =

5.36 m/s