FINAL EXAM

Print your name and section clearly on all nine pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8.5" x 11" sheet and no other references. The exam lasts exactly two hours.

SOLUTION KEY

(Do not write below)
SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

Problem 5: __________

Problem 6: __________

Problem 7: __________

Problem 8: __________

TOTAL: __________

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

$k = 8.99 \times 10^9 Nm^2 C^{-2}$

$\mu_0 = 4\pi \times 10^{-7} TA^{-1} m^{-1}$

electron mass $m_e = 9.11 \times 10^{-31}$ kg

elementary charge $e = 1.60 \times 10^{-19}$ C

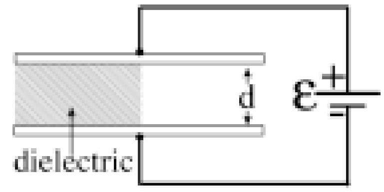
speed of light $c = 3.00 \times 10^8$ m/s

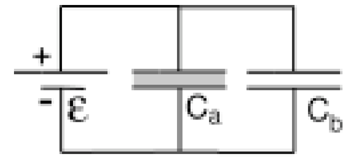
1 eV = 1.602×10^{-19} J

1 T = 1 $NA^{-1}m^{-1} = 10^4$ G

PROBLEM 1

A parallel plate capacitor of plate separation d is connected to an ideal battery of constant emf $\boldsymbol{\varepsilon}$. When the space between the plates is empty, the capacitance is C_0. When a dielectric slab of thickness d is inserted between the plates occupying 1/2 of the space between the plates as shown, the capacitance increases to $3C_0$. Neglect fringing effects.





a) What is the dielectric constant κ of the slab? *(5 pts)*

$C = C_a + C_b = 3C_0$, $Ca = \kappa(1/2)C_0$, $C_b = (1/2)C_0 \Rightarrow (\kappa+1)(1/2)C_0 = 3C_0$
$\Rightarrow \kappa+1 = 6 \Rightarrow \kappa = 5$

$\boxed{5}$

b) What is the electric energy stored in the capacitor before insertion of the dielectric slab? *(5 pts)*

$U = (1/2)C_0\boldsymbol{\varepsilon}^2$

$\boxed{(1/2)C_0\boldsymbol{\varepsilon}^2}$

c) What is the change, ΔU, in the electric energy stored in the capacitor due to the insertion of the dielectric slab? *(5 pts)*

$\Delta U = U_f - U_0 = (1/2)C_f\boldsymbol{\varepsilon}^2 - (1/2)C_0\boldsymbol{\varepsilon}^2 = (1/2)\,\boldsymbol{\varepsilon}^2(3C_0 - C_0) = (1/2)\,\boldsymbol{\varepsilon}^2(2C_0) = C_0\boldsymbol{\varepsilon}^2$

$\boxed{C_0\boldsymbol{\varepsilon}^2}$

d) How much work, $W_{battery}$, does the battery do when the dielectric slab is inserted? *(5 pts)*

$W_b = \boldsymbol{\varepsilon}\Delta q = \boldsymbol{\varepsilon}\,(q_f - q_0)$. $q_0 = C_0\boldsymbol{\varepsilon}$, $q = C_f\boldsymbol{\varepsilon} = 3C_0\boldsymbol{\varepsilon} \Rightarrow \Delta q = 2C_0\boldsymbol{\varepsilon} \Rightarrow W_b = \boldsymbol{\varepsilon}(2C_0\boldsymbol{\varepsilon}) = 2C_0\boldsymbol{\varepsilon}^2$

$\boxed{2C_0\boldsymbol{\varepsilon}^2}$

e) When the capacitor with dielectric is fully charged, the battery is suddenly replaced with a resistor R. How long does it take for the charge to reduce to 13.5% of its full charge? *(5 pts)*

$$e^{-\frac{t}{RC}} = .135 \Rightarrow \frac{-t}{RC_f} = \ln(.135) = -2 \Rightarrow t = 2RC_f = 2R(3C_0) =$$

$\boxed{6RC_0}$

PROBLEM 2

A 25.0 W point source of sound waves of frequency 1200.0 Hz is moving right with a speed of 30.0 m/s relative to the air. Ahead of it is a reflecting surface moving left with a speed of 66.0 m/s relative to the air. The speed of sound in air is 329 m/s.

a) How many decibels of unreflected sound are received by a listener traveling along with the source a fixed distance of 5.00 m away? *(5 pts.)*

$$I = \frac{P}{4\pi R^2} = \frac{25.0W}{4\pi(5.00m)^2} = 0.0795\frac{W}{m^2} \Rightarrow \beta = 10\log\left(\frac{0.0795W/m^2}{10^{-12}W/m^2}\right) = 109db$$

109 db

b) What wavelength of sound is emitted towards the reflector by the source? *(5 pts.)*

$$\lambda = \frac{v}{f}\left(1 - \frac{u}{v}\right) = \frac{329\,m/s}{1200Hz}\left(1 - \frac{30\,m/s}{329\,m/s}\right) =$$

0.249 m

c) What is the frequency of sound received by the reflector? *(5 pts.)*

$$f' = f\left(\frac{v + v_r}{v - v_s}\right) = 1200Hz\left(\frac{329\,m/s + 66\,m/s}{329\,m/s - 30\,m/s}\right) =$$

1585 Hz

d) What is the wavelength of the reflected waves? *(5 pts.)*

$\lambda' = (v - v_r)/f' = (329\text{ m/s} - 66\text{ m/s})/(1585\text{ Hz}) =$

0.166 m

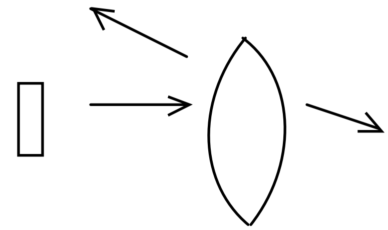
e) What is the frequency of the reflected sound received back at the source? *(5 pts.)*

$$f_r = f'\left(\frac{v + v_s}{v - v_r}\right) = 1585Hz\left(\frac{329\,m/s + 30\,m/s}{329\,m/s - 66\,m/s}\right) =$$

2164 Hz

PROBLEM 3

A converging thin lens made of glass (n=1.40) has a radius of curvature R=20.0 cm for both surfaces. An object of height 23.3 cm is placed 35.0 cm in front of the lens. It forms two images, one due to refraction (rays passing though the lens) and one due to partial reflection from the first surface encountered by the rays (acting as a mirror).



a) What is the distance of the image from the reflection from the front of the lens? *(5 pts)*

Focal length for convex mirror $f_{mirror} = -R/2 = -10.0$ cm
Relation between distance to object p, distance to image q, and focal length f is
$1/q + 1/p = 1/f$, so $q = 1/(1/f-1/p)=1/(-1/(10.0\text{cm})–1/(35.0\text{cm})) = -7.78$ cm

7.78 cm

b) What is the distance of the refracted image from the lens? *(6 pts)*

Focal length of lens found from lens equation,
$1/f_{lens} = (n-1)(1/R_1+1/R_2)=(0.40)(1/(20.0\text{cm})+1/(20.0\text{cm}))$, so $f_{lens} = 25.0$ cm
$1/q + 1/p = 1/f$, so $q = 1/(1/f-1/p)=1/(1/(25.0\text{cm})–1/(35.0\text{cm})) = 87.5$ cm

87.5 cm

c) What is the size of the reflected image from the front surface of the lens? Is it **real** or **virtual** (circle one)? Is it **right side up** or **upside down** (circle one)? *(7 pts)*

$M = -q/p = (7.78\text{ cm})/(35.0\text{ cm}) = 0.222$
So image is right side up, and it has height (0.222)(23.3cm) = 5.17 cm
Image is behind mirror, so it is virtual.

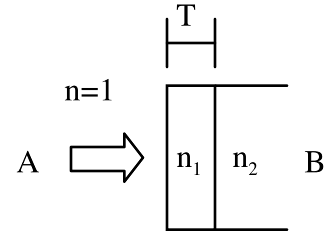
5.17 cm

d) What is the size of the refracted image? Is it **real** or **virtual** (circle one)? Is it **right side up** or **upside down** (circle one)? *(7 pts)*

$M=-q/p = (-87.5\text{ cm})/(35\text{ cm}) = -2.5$.
So image is upside down, and has height (2.5)(23.3 cm) = 58.25 cm.
Light rays intersect at image, so it is real.

58.25 cm

PROBLEM 4



Light of vacuum wavelength 597 nm is normally incident on a flat glass plate which has on its front surface a coating with refractive index n_1=1.30. The refractive index of the glass is n_2=1.40.

a) What is the thickness, T, of the thinnest coating layer that causes destructive interference in the reflected beam, suppressing reflection of the light from the glass plate at point A? *(5 pts.)*

Both interfaces cause π phase shift, so get first minimum in reflected intensity when $2n_1T = \lambda/2$, or $T=\lambda/(4n_1) = (5.97\times10^{-7}\text{ m})/(4\times1.30) = 1.148\times10^{-7}$ m

$\boxed{0.115\ \mu}$

b) What is the wavelength of the light from part a) in the glass (refractive index n_2=1.40)? *(5 pts.)*

$\lambda_{glass} = \lambda/n_2$ = (597 nm)/(1.40) = 426.4 nm

$\boxed{426\text{ nm}}$

c) What is the frequency of the light from part a) in the glass (refractive index n_2=1.40)? *(5 pts.)*

Frequency is same in all the media, so $f = c/\lambda = (3.00\times10^{8}\text{ m/s})/(5.97\times10^{-7}\text{ m})$ =

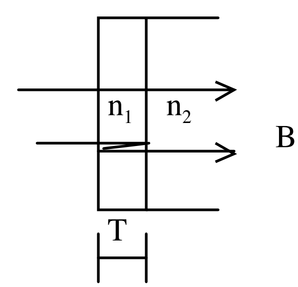
$\boxed{5.03\times10^{14}\text{ Hz}}$

d) Assume that the refractive index of the glass and the coating do not depend on the light frequency. What is the next-shortest vacuum wavelength for which there is destructive interference in reflection from the glass plate? *(5 pts.)*

Second minimum for new wavelength λ´ occurs when $2n_1T = 3\lambda'/2$, or
$\lambda'=4n_1T/3 = 4(1.30)(1.148\times10^{-7}\text{ m})/3$ =

$\boxed{199\text{ nm}}$

e) What is the minimum coating thickness T so that observer B on the right sees maximum constructive interference between the two paths shown for red light with a wavelength in vacuum of λ_{red} = 751 nm? *(5 pts.)*

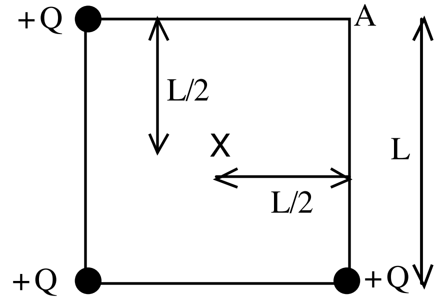


First reflection has phase shift of π, while second reflection off front surface has no phase shift. So smallest thickness with constructive interference has $2n_1T = \lambda/2$, or $T=\lambda/(4n_1)$ = (751 nm)/(4×1.30) = 144 nm

$\boxed{0.144\ \mu}$

PROBLEM 5

Three identical point charges $+Q = 1.57\times10^{-6}$ C are placed at the corners of a square with sides of length L = 2.23 m, as shown. There is no charge at one of the corners (marked A on the figure).



a. What is the magnitude of the electric field at the point X *in the center of the square*? *(5 pts.)*

Field contributions from charges at upper left and lower right corners are equal and opposite, so net comes entirely fromcontribution from lower left corner, which has magnitude

$kQ/R^2 = kQ/\left(L/\sqrt{2}\right)^2 = 2kQ/L^2 =$

$= (2)(8.99\times10^9 \text{ N m}^2 \text{ C}^{-2})(1.57\times10^{-6} \text{ C})/(2.23 \text{ m})^2 = 5676$ N/C

$\boxed{5680 \text{ N/C}}$

b. What is the electric potential difference, $V = V_A - V_X$ between the point A at the empty corner of the square and the point X at the center of the square? *(5 pts.)*

$V_A = 2kQ/L + kQ/\left(\sqrt{2}L\right)$, $V_X = 3kQ/\left(L/\sqrt{2}\right)$, so $V = V_A - V_X = (kQ/L)\left(2-5/\sqrt{2}\right)$

$= (8.99\times10^9 \text{ N m}^2 \text{ C}^{-2})(1.57\times10^{-6} \text{ C})/(2.23 \text{ m}))(2-5/1.414) =$

$\boxed{-9720 \text{ V}}$

c. What is the electric potential energy of this charge configuration? *(5 pts.)*

The electric potential energy, U_1, is

$$U_1 = \frac{kQ^2}{L} + \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L} = \left(2+1/\sqrt{2}\right)\frac{kQ^2}{L} = \left(2+1/\sqrt{2}\right)\frac{\left(8.99\times10^9 Nm^2C^{-2}\right)\left(1.57\times10^{-6}C\right)^2}{2.23m} =$$

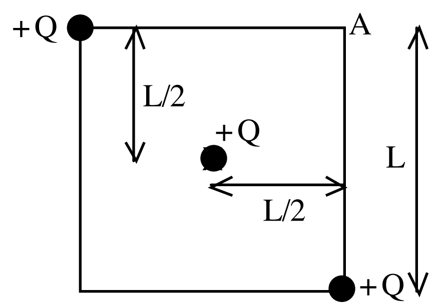
$\boxed{0.0269 \text{ J}}$

d. What is the capacitance of the charge configuration in parts (a-c)? *(5 pts.)*

$U=Q^2/2C \Rightarrow C=Q^2/2U= (1.57\times10^{-6} \text{ C})^2/(2\times0.0269 \text{ J}) = 4.58\times10^{-11}$ F

$\boxed{45.8 \text{ pF}}$

e. How much work must be done (against the electric force) to move the charge at the lower left corner to the center of the square at point X, resulting in the configuration shown? *(5 pts.)*



Electric potential energy of this new configuration is

$$U_1 = \frac{2kQ^2}{\left(L/\sqrt{2}\right)} + \frac{kQ^2}{\sqrt{2}L} = \left(2\sqrt{2}+\sqrt{2}/2\right)\frac{kQ^2}{L} = \frac{5}{2}\sqrt{2}\frac{kQ^2}{L}$$

$=(2.5)(1.414)(8.99\times10^9 \text{ N m}^2 \text{ C}^{-2})(1.57\times10^{-6} \text{ C})^2/(2.23 \text{ m}) =$ 0.351 V.

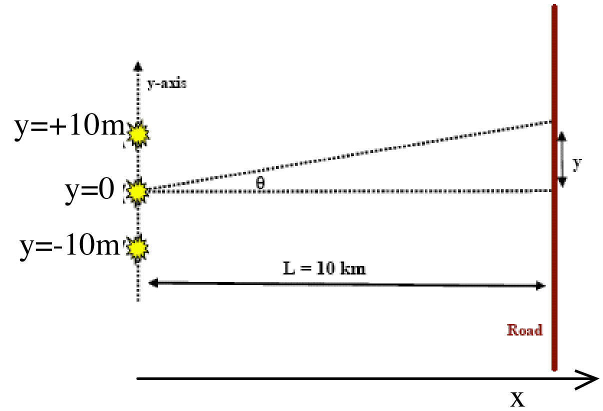
So work done = 0.0351 J – 0.0269 J =

$\boxed{0.0082 \text{ J}}$

First Name: ____________________ Last Name: ____________________ Section: __________ ■

PROBLEM 6

Three identical radio broadcast towers lie along the y-axis at y=-10.0m, y=0, and y=+10.0m. All three towers broadcast in phase at a frequency of 123 MHz with equal intensities. There is a road parallel to the y axis a distance L = 10.0 km from the radio towers, as shown. An observer walking along the road with a radio tuned to 123 MHz receives an intensity I_0 at y = 0 (*i.e.* $I(\theta=0) = I_0$). (L >> d, so use the small angle approximation.)

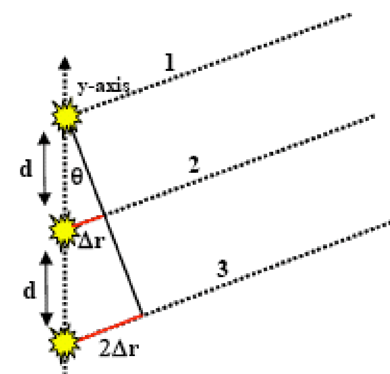


a) What is the wavelength of the radio wave emitted by one of the towers? *(5 pts.)*

$\lambda = c/f = (3.00\times10^8 \text{ m/s})/(1.23\times10^8 \text{ Hz}) =$

$\boxed{2.44 \text{ m}}$

b) What is the difference in the path lengths for the light from the middle tower (at y=0) and from the tower at y=-10.0m when the observer is at y=333m up the road from y=0? *(5 pts.)*

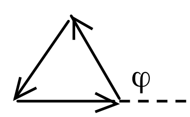


$\Delta r = d\sin\theta$, with $\tan\theta = y/L$. When theta is small, $\sin\theta \approx \tan\theta \approx \theta$, so
$\Delta r = dy/L = (10.0 \text{ m})(333 \text{ m})/(10^4 \text{ m}) = 0.333 \text{ m}$

$\boxed{0.333 \text{ m}}$

c) If the observer starts at y = 0 and walks along the road in the positive y direction, at what distance **y** (**in km**) will the intensity of the radio signal first be zero due to interference between the three towers (*i.e.* what is the smallest distance y at which total destructive interference occurs)? *(5 pts.)*

Smallest y with totally destructive interference is when the three phasors shown add to zero, or $\varphi = 2\pi/3$, where phase difference $\varphi = 2\pi\Delta r/\lambda = (2\pi)(dy/L)/\lambda$.
So $y = L\lambda/(3d) = (10^4 \text{ m})(2.44 \text{ m})/(3\times10 \text{ m}) = 813 \text{ m}$



$\boxed{0.813 \text{ km}}$

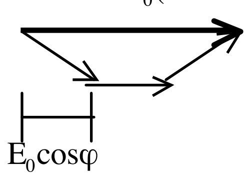
d) If the observer starts at y = 0 and walks along the road in the positive y direction, what is the smallest distance y (in km) at which all three sources interfere constructively (not counting y=0)? *(5 pts.)*

All three sources interfere constructively when $d\sin\theta = m\lambda$, for m an integer. Since $\theta \approx y/L$, smallest y where sources interfere constructively is given by
$y = \lambda L/d = (2.44 \text{ m})(10^4 \text{ m})/(10 \text{ m}) = 2440 \text{ m}$

$\boxed{2.44 \text{ km}}$

e) What fraction of I_0 is the intensity of the signal when the observer is on the road at y=+0.4067 km? *(5 pts.)*

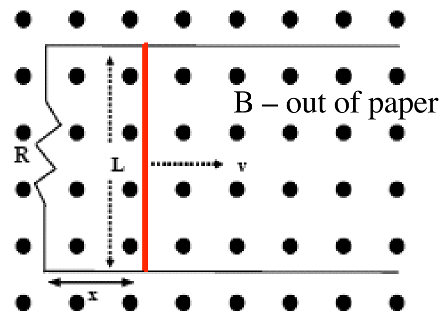
Phase difference between successive sources is $\varphi = 2\pi dy/(\lambda L) = 1.047$ rad, so magnitude of resultant E is $E = E_0(1+2\cos\varphi)$, versus $3E_0$ for $\varphi=0$ So $I/I_0 = (1+2\cos\varphi)^2/9 =$



$\boxed{0.445}$

PROBLEM 7

A moveable bar of length L=0.25 m is pulled along two frictionless conducting rails at constant speed v=5.0 m/s to the right, as shown. The entire system is immersed in a uniform constant magnetic field (*out of the paper*) with magnitude B_0 = 2.0T. The resistor has resistance R=2.1 Ω. At time t=0 the bar is at x=0.



a) What is the magnitude of ε, the induced EMF in the loop, when the bar is moving at constant speed? *(5 pts.)*

The magnetic flux through the loop at time t is given by
$\Phi_B = B_0A = B_0xL = B_0Lvt$,
and from Faraday's Law of induction the induced EMF is
$\varepsilon = -\dfrac{d\Phi_B}{dt}$ = $-B_0Lv$ (independent of time),
so $|\varepsilon| = B_0Lv$ = (2.0 T)(0.25 m)(5.0 m/s) =

$\boxed{2.5\ \text{V}}$

b) What is the magnitude of the current in the resistor? *(5 pts.)*

I = (2.5 V)/(2.1 Ω) = 1.19 A

$\boxed{1.2\ \text{A}}$

c) What is the magnitude of the net magnetic force (in N) on the rod? *(5 pts.)*

The magnitude of the magnetic force on the rod is
$F_B = ILB_0$ = (1.2 A)(0.25 m)(2.0 T) = 0.59 N

$\boxed{0.59\ \text{N}}$

d) If the resistor R is the filament of a 50.0 Watt light bulb, at what speed v (in m/s) must one pull the rod in order to supply 50.0 W to the light? *(5 pts.)*

Power supplied to the light is $P = I^2R = (B_0Lv)^2/R$, so
$v=(PR)^{1/2}/(B_0L) = ((50.0\ \text{W} \times 2.1\ \Omega)^{1/2}/(2.0\ \text{T} \times 0.25\ \text{m})$ = 20.49 m/s

$\boxed{20\ \text{m/s}}$

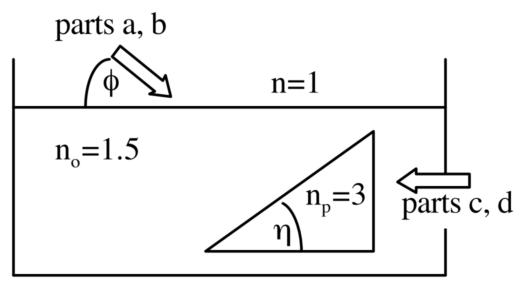
e) An electron (mass 9.11×10^{-31} kg, charge 1.60×10^{-19} C) enters the 2.0 T magnetic field with a velocity of 3600 m/s in the +x direction. What is the cyclotron radius of the electron's subsequent motion?

Cyclotron radius = mv/eB = $(9.11 \times 10^{-31}$ kg)(3600 m/s)/((1.60×10^{-19} C)(2.0 T)) = 1.0×10^{-8} m

$\boxed{1.0\times10^{-8}\ \text{m}}$

PROBLEM 8

A prism shaped like a right triangle with index of refraction n_p=3.0 is resting in a container of oil with refractive index n_o=1.5. Light is incident from the air at an angle ϕ above the horizontal.



a) What is the angle ϕ at which the light reflected at the oil surface back into the air is completely polarized? *(5 pts.)*

Angle of incidence θ_{in} is $\theta_{in} = \frac{\pi}{2} - \phi$. Light is completely polarized at Brewster's angle, where

$\tan\theta_{in} = (n_o/n)$. So $\phi = \frac{\pi}{2} - \tan^{-1}(n_o/n) = \frac{\pi}{2} - \tan^{-1}(1.5) =$

$\boxed{0.59 \text{ rad}}$

b) If the angle of the prism is η=30.0°, what is the value of the angle ϕ at which the path of a light ray reflected from the top surface of the prism retraces the same path it took on the way in towards the prism? *(5 pts.)*

Path retraces if the rays are incident normally on the prism. This happens if they are at an angle η from the vertical after refraction from the oil surface. Therefore,

$\sin\left(\frac{\pi}{2} - \phi\right) = n_o \sin\eta$, or $\phi = \cos^{-1}(n_o\sin\eta) = \cos^{-1}((1.5)(1/2)) =$

$\boxed{0.72 \text{ rad}}$

c) When light is incident horizontally on the prism from the right, what is the largest angle η for which no light is transmitted through the top surface of the prism? *(5 pts.)*

Angle between light and normal of top surface is $\frac{\pi}{2} - \eta$. Smallest value of this angle at which total internal reflection occurs is determined by $n_p\sin(\frac{\pi}{2} - \eta) = n_o$, so

$\eta = \cos^{-1}(n_o/n_p) = \cos^{-1}(1.5/3) = 1.047$ rad

$\boxed{1.0 \text{ rad}}$

d) When light is incident horizontally on the prism from the right, what is the angle η for which the light reflected from the top of the prism goes down vertically? *(5 pts.)*

Angle between light and normal of top surface is $\frac{\pi}{2} - \eta$. Beam goes down vertically when angle of reflection is η. Since angle of incidence equals the angle of reflection, the beam goes down vertically when 2η=π/2, or η=π/4 rad.

$\boxed{45°}$

e) The top of the prism is a depth 0.227 m below the oil's surface. What is the apparent depth of the top of the prism, as viewed from directly overhead in the air?

For this flat refracting surface, distance to image q is related to distance to object p via
$q = -(1/n_o)p = (-1/1.5)(0.227 \text{ m}) = 0.15$ m.

$\boxed{0.15 \text{ m}}$