Electric Field in a Conductor

<u>Conductor</u> = charges are free to move

<u>Insulator</u> = charges are fixed

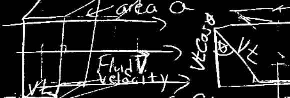
If Electric Field in a Conductor ⇒ Force on charges ⇒ $F = ma$ ⇒ Charges move

Moving charges = Current

<u>If</u> No Current in a Conductor

Net Field = 0

Flux Fluid flowing in one direction

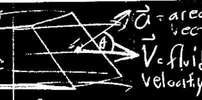


Flux = Volume of fluid thru square per unit time

In t seconds a volume of fluid Vta goes thru frame

Volume of Fluid per time

$$\frac{Vta}{t} = Va = Flux$$



Define vector $\vec{a}$
has magnitude of area of frame and direction perpendicular to frame

Volume of fluid thru frame $= Vt\cos\theta\, a$
$= \vec{V}\cdot\vec{a}$ = Flux

Define Electric Flux thru surface with area vector $\vec{a}$
$= \vec{E}\cdot\vec{a} = \Phi$ = Scalar

$\Phi = Ea\cos\theta$

Different surface shapes
Different field shapes

Balloon:

Divide surface into patches

Each patch j with area vector $\vec{a}_j$ and field value (assume constant) $\vec{E}_j \Rightarrow$ get flux $\Phi_j = \vec{E}\cdot\vec{a}_j$

Sum up fluxes from all patches = Total Flux Φ through whole surface

$$\Phi = \sum_{\text{all } j} \vec{E}\cdot\vec{a}_j = \int_{\text{surface}} \vec{E}\cdot d\vec{A}$$

patches small

Gauss Law

Simplest case
single point charge
at center of a sphere
of radius r

$E = \frac{k q_h}{r^2}$ and points along radius vector

Area vector is perpendicular to surface → points along radius vector

Electric field has constant value along a surface at constant distance r = sphere

$\Rightarrow$ Flux $= \int \vec{E} \cdot d\vec{A}$

$= \int E\,dA = E \int dA$

$= E\, 4\pi r^2$

$= \frac{k q_h}{r^2} \cdot 4\pi r^2$

$\Rightarrow \Phi = 4\pi k q_h$, q_h is charge enclosed by sphere

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Flux through any closed surface = $4\pi k$ × charge enclosed by that surface = Gauss' Law: $\Phi = 4\pi k Q$

$\Phi_c = \oint \vec{E} \cdot d\vec{A} = \oint E_n\, dA$

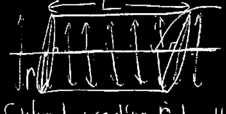
- Φ_c ↑ Flux thru closed surface
- $\oint$ ↑ Integral over closed surface
- E_n ↑ Field normal to surface at each point

$$I_c = \oint \vec{E}\cdot d\vec{A} = 4\pi K Q_{encl}$$

Field of an infinite line charge

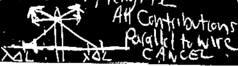
Charge per Length = λ

λ = Coulombs/meter



Cylinder radius r, length L

Ends:



only uncancelled contributions to field are perpendicular to wire: Flux thru ends.

only Field is Parallel to end surface ⇒ its perpendicular to area vector ⇒ $\vec{E}\cdot\vec{a} = 0$ thru ends ⇒ No Flux thru ends

only Flux thru side = area of cylinder side • Electric Field E is constant = E_r over surface and its perpendicular to surface

E_r = radial field

$$\oint \vec{E}\, d\vec{A} = \int E_r\, dA = E_r \int dA = E_r\, 2\pi r L \overset{Gauss}{=} 4\pi K Q$$

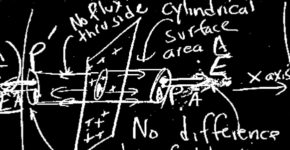
$$Q = \lambda L$$

$2\pi r L E_r = 4\pi k \lambda L$

$E_r = \frac{2k\lambda}{r} = \frac{1}{2\pi\epsilon_0}\left(\frac{\lambda}{r}\right)$

Infinite sheet of Charge, surface Charge density σ

Charge in area A is σA



Field must be perpendicular to surface because parallel components cancel (c.f. line charge). Field points

Total flux = $\int E \cdot dA$

$= 2AE_p$ by Gauss $= 4\pi k Q_{encl} = 4\pi k \sigma A$

$2AE_p = 4\pi k \sigma A$

$$E_p = 2\pi k \sigma$$

$$\vec{E}_p = 2\pi k \sigma \hat{x}$$

$\vec{E}_{p'} = -2\pi K \sigma \hat{x}$

Field strength is independent of distance from surface

Gauss: Any charge on conductor lies on surface. No current: field inside = 0 ⇒ Flux through any closed surface inside = 0

Surround any charge inside with surface ⇒ gives $\vec{E}$ unless charge = 0, but $\vec{E} = 0$ in conductor ✓