Course Pages
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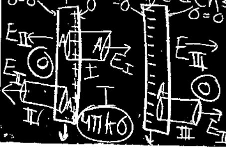
Field is ⊥ to plates
⇒ No Flux thru cylinder
Sides: only Flux thru
cylinder ends of area A

2 oppositely charged (∞)
Parallel Conducting Plates
Surface charge density σ



3 Gaussian Cylindrical Closed surfaces

Inside a Conductor (no current)

$E = 0$

Apply Gauss to Surfaces (Cylinders)

I: ϕ thru sides, ϕ thru end in Conductor
only Flux from end between plates

$E_I A = \Phi_I = 4\pi K Q_{encl}$

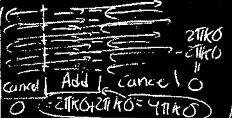
$Q_{encl} = \sigma A \Rightarrow E_I A = 4\pi K \sigma A$

$\boxed{E_I = 4\pi K \sigma}$ field btw Plates

II (III same): Sides = 0, end in plate = 0 $\Rightarrow \Phi_{II} = E_{II} A = 4\pi k Q$

$Q = 0 \Rightarrow E_{II} A = 0 \Rightarrow E_{II} = 0$

$\Rightarrow$ also $E_{III} = 0$



Field btw Plates

$E = 4\pi k\sigma = \sigma / \epsilon_0$

Potential Energy

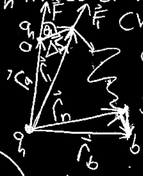
Electric Forces are Conservative

Work = Force x distance

Conservative = Path independent

Moving charge q_h' under force of Stationary Charge q_h

q_h' moves $a \to b$. force $\vec{F}$ Changes direction + magnitude.

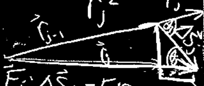


Divide path into pieces $\Delta\vec{S}_j$ where Force $= \vec{F}_j$

Work done to move some distance $\Delta\vec{S}_j$

$W_j = \vec{F}_j \cdot \Delta\vec{S}_j$

$$F_j = \frac{k q_h q_h'}{r_j^2}$$



$\vec{F}_j \cdot \Delta\vec{S}_j = F_j \Delta S_j \cos\theta_j$

$\Delta S_j \cos\theta = r_j - r_{j-1} = \Delta r_j$

$W_j = \vec{F}_j \Delta\vec{S}_j = F_j \Delta S_j \cos\theta = F_j \Delta r_j$

pieces → very small, sum over

$W_j \Rightarrow$ integral ⇒

$$W = \int_{\vec{r}_a}^{\vec{r}_b} \vec{F} d\vec{s} = \int_{r_a}^{r_b} F ds \cos\theta$$

$$= \int_{r_a}^{r_b} F dr = \int_{r_a}^{r_b} \left(\frac{k q_h q_h'}{r^2}\right) dr$$

$$= k q_h q_h' \int_{r_a}^{r_b} \frac{dr}{r^2} = k q_h q_h' \left(-\frac{1}{r}\right)_{r_a}^{r_b}$$

$$W = k q_h q_h' \left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

only depends on beginning and end position, not on path.

2 charges q_{h_1}, q_{h_2} far apart

bring together distance r_{12}

$q_{h_1} \; \infty \; q_{h_2} \Rightarrow q_1 \xrightarrow{r_{12}} q_{h_2}$

$$W = k q_{h_1} q_{h_2} \left(\frac{1}{r_{12}} - \frac{1}{\infty}\right) = \frac{k q_{h_1} q_{h_2}}{r_{12}}$$

Potential Energy of a system of 2 charges

$$U = \frac{k q_{h_1} q_{h_2}}{r_{12}}$$

Bring in a 3rd charge (while q_{h_1}, q_{h_2} fixed apart by r_{12}) to distance r_{13} from q_{h_1} and r_{23} from q_{h_2}

Superposition: bring q_{h3} in near q_1, calculate energy then bring near q_2 calculate energy and add.

$$W_3 = k\frac{q_{h1} q_{h3}}{r_{13}} + k\frac{q_{h2} q_{h3}}{r_{23}}$$

Add to original Work q_1, q_2:

$$U_{\text{3 charges TOTAL}} = k\frac{q_{h1} q_{h2}}{r_{12}} + k\frac{q_{h1} q_{h3}}{r_{13}} + k\frac{q_{h2} q_{h3}}{r_{23}}$$

Total Potential Energy of 3 Charges

Note: No change if Swap 1⇔2, 2⇔3, 3⇔1

U is a property of the arrangement of charges (assembly indep.)

Bring a test charge q_h into a region of <u>many</u> other fixed charges

Define a potential energy per unit charge

= The <u>Potential</u>

$$= V = \frac{U}{q_h} \quad \text{or} \quad U = q_h V$$

Units = $\frac{\text{Joules}}{\text{Coulomb}}$ = <u>Volt</u>

W to move charge a → b = Difference in U at a & b

$$W_{a\to b} = U_a - U_b$$

$$\Rightarrow \frac{W_{a\to b}}{q_h} = \frac{U_a}{q_h} - \frac{U_b}{q_h} = V_a - V_b$$

V_a = Potential at point a
V_b = Potential at point b
$V_a - V_b$ = Potential difference between a and b
$= V_{ab}$
For any # of Charges q_i
Bringing a charge q_0

$$U = k q_0 \sum \frac{q_i}{r_{i0}} \Rightarrow$$

Potential Energy of q_0 at r_0

$$V = k \sum \frac{q_i}{r_{i0}} \quad \text{Def'n}$$

Potential at point r_0

Now look test in an Electric field
Work to move q_0 $d\vec{s}$: $dW = \vec{F} \cdot d\vec{s}$
$= q_0 \vec{E} d\vec{s}$ since $\Delta U = -W$

$$= -\int_a^b \vec{F} d\vec{s} = \text{Work to move from point } a \to b$$

$$= -\int_a^b q_0 \vec{E} d\vec{s} = -q_0 \int_a^b \vec{E} d\vec{s}$$

$$V_b - V_a = \frac{U_b - U_a}{q_0} = -\int_a^b \vec{E} \cdot d\vec{s}$$

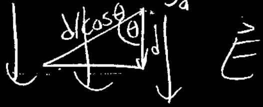
$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$ Electric field along path

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$= -\int_a^b E \cos\theta \, ds$$

$$= -\int_a^b E \, ds$$

$$V_b - V_a = -E\int_a^b ds = -Ed$$



$$V_b - V_a = -\int_a^b \vec{E}\, ds$$

$$= -\int_a^b E\cos\theta\, ds$$

$$= -E\cos\theta \int_a^b ds$$

$$= -E\cos\theta \cdot \frac{d}{\cos\theta} = -Ed \checkmark$$