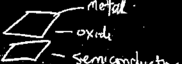
UPS tutoring 2321 Sterling

M	9:55	1:20	2:25
T	11		
W	9:55	1:20	2:25
R	11		
F	9:55	1:20	

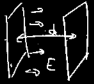
CMOS
logic computers



Capacitors

Potential

last time:
uniform electric field



$V = Ed$

Charged conducting sphere of radius R $\Rightarrow V = \frac{kq}{r}$ (outside)

Sphere

field outside sphere is same as for point charge

$\vec{E} = \frac{kq}{r^2}\hat{r}$

Inside sphere

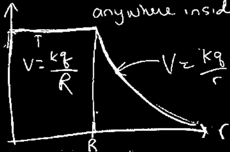
$\vec{E} = 0$

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

($\vec{E}$: zero)

$V_b = V_a$

anywhere inside



outside the sphere

$r > R$

charged sphere yields same potential as point charge at $r = 0$

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inside sphere, field is <u>zero</u>

potential is <u>constant</u>

(can be large)

Given potential $V(r)$, what is field?

1^{st}

$E = \sigma/\epsilon_0$ (from Gauss' law)

rewrite it in terms of voltage V

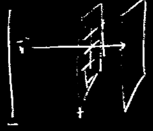
move charge q from plate A to plate B.

$W = \vec{F}\cdot\vec{d} = qEd = \sigma q d/\epsilon_0$

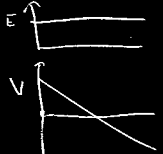
Potential $V = W/q = V_A - V_B = V_{AB} = \frac{W}{q} = \frac{\sigma}{\epsilon_0}d = Ed.$

So in terms of V

$E = V/d$ parallel plates



constant field ⟹ V is linear in position



Units.

given charge q.
what is change in Potential energy when accelerated by a voltage V_{ab}?

$$\Delta U = q(V_{ab})$$

for $\begin{cases} q = e = 1.60 \times 10^{-19}\ C \\ V_{ab} = 1\ Volt \end{cases}$

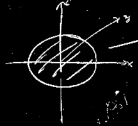
then $\Delta U = qV = (1.6 \times 10^{-19} C)(1V)$
$= 1.6 \times 10^{-19}\ J$

$$\boxed{1\ eV = 1.6 \times 10^{-19}\ J}$$ (Unit of energy)

Calculate potential for continuous Charge distribution

e.g. disk in x-y plane

potential along axis



→ divide the disk into rings

radius = s
width = ds.

total potential
is sum of potentials
from all the rings

find potential from 1 ring.
area of ring at radius s
with width ds

$= 2\pi s\, ds$

charge of ring = $\sigma\, 2\pi s\, ds$

distance from ring to point in question $= \sqrt{s^2+z^2}$

So ring contribution is $k(2\pi\sigma s\, ds) / \sqrt{s^2+z^2}$

add contributions from rings..

$$V = \int_0^R k\sigma \frac{2\pi s}{\sqrt{s^2+z^2}}\, ds = 2\pi k\sigma \int_0^R \frac{s\, ds}{\sqrt{s^2+z^2}}$$

$$= 2\pi k\sigma \sqrt{z^2+s^2}\,\Big]_0^R$$

$$V(z) = 2\pi k\sigma\left[\sqrt{z^2+R^2} - z\right]$$

Now calculate $\vec{E}(\vec{r})$ given $V(\vec{r})$

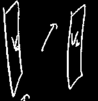
recall

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s}$$

$$\Rightarrow \vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

1st example

parallel plates

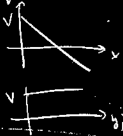


$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

define $E_{\parallel} = E\cos\theta$ ← angle between E and ds.

$$V_b - V_a = -\int_a^b E_{\parallel}\, ds.$$

$$E_{\parallel} = -\frac{dV}{ds}$$ = rate of change of potential in direction ds.



$E_x \neq 0$

$E_y = 0$

$E_z = 0$

$$E_x = -\frac{\partial V}{\partial x},\quad E_y = -\frac{\partial V}{\partial y},\quad E_z = -\frac{\partial V}{\partial z}$$

$$\text{so } \vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -\vec{\nabla} V$$

disk: potential, $V = 2\pi K\sigma\left(\sqrt{z^2+R^2} - z\right)$

$$E_x = E_y = 0$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z} 2\pi k\sigma\left(\sqrt{z^2+R^2} - z\right) = 2\pi k\sigma\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$