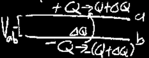
Capacitor: Capacitance C



$Q = C V_{ab}$

$\Delta W = V_{ab}\Delta Q = \frac{Q}{C}\Delta Q$

Charge capacitor from $Q=0$ to $Q=Q_f$ (final)

Requires $W = Q_{AV} Q_f / C$

Q_{AV} = average charge on plates during movement of charge

$Q_{AV} = \frac{1}{2}(0+Q_f) = \frac{Q_f}{2}$

$W = \frac{Q_f}{2}\frac{Q_f}{C} = \frac{Q_f^2}{2C}$

= Work = energy stored

Let q_h be charge at some instant, $V = q_h/C$

Move dq_h, $dW = V\,dq_h = \frac{q_h}{C}dq_h$. Charge from $0 \to Q$

$W = \int_0^Q \frac{q_h}{C}dq_h = \boxed{\frac{Q^2}{2C}}$

$\boxed{U = Q^2/2C}$

$Q = CV \Rightarrow U = \frac{(CV)^2}{2C} =$

$\boxed{U = \frac{1}{2}CV^2}$

or

$U = Q(CV)/2C$

$= \boxed{\frac{1}{2}QV = U}$

Parallel Plate Cap:

$C = \frac{\epsilon_0 A}{\ell}$ (A ← plate area, ℓ ← plate separation)

Also: $E = V/\ell \Rightarrow V = E\ell$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{\ell}\right)(E\ell)^2$

$= \frac{1}{2}\epsilon_0 E^2 A\ell = \frac{1}{2}\epsilon_0 E^2 \cdot Vol$

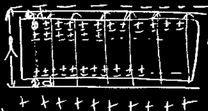
Vol = Volume between Plates
= Volume occupied by E
⇒ Energy Stored in E
$= \frac{1}{2}\epsilon_0 E^2 \cdot$ Volume occupied

Energy density $U \doteq \frac{1}{2}\epsilon_0 E^2$ of Electric Field

Slab of dielectric (insulating, polarizable) material b/w plates of Capacitor.

Molecules can't move but can orient.

$C = \frac{\epsilon_0 A}{\ell}$ ← plate sep closer → increase C



layer of charge now a distance $\ll \ell$ away from plate
→ Capacitance increases.

Ratio of empty to completely full w/dielectric for a capacitor $K = C/C_0$

C = new capacitance w/ dielectric
C_0 = original cap. w/o dielectric
$\boxed{K = C/C_0}$. for Vacuum: $K=1$

Charged Capacitor, disconnected from a battery, put in dielectric
⇒ Q unchanged, but $V = Q/C$ decreases since $\boxed{C = KC_0}$

$V_0 = \frac{Q}{C_0}$, $V = \frac{Q}{KC_0} = \frac{1}{K} V_0$

$\boxed{K > 1}$ → Voltage decreases

Cap: $E = \frac{V}{\ell}$ ← plate sep.

Dielectric → V decreases
⇒ What about E?

$\boxed{E_0 = \sigma/\epsilon_0}$ and charge $\sigma = \frac{Q}{A}$ has not changed.

Some of charge on plates is cancelled by induced charge on dielectric
= layer of charge on dielectric

⇒ Induced surface charge density on dielectric = σ_i

$$\boxed{E = \frac{\sigma - \sigma_i}{\epsilon_0}} = \frac{V}{\ell}$$

$$K = \frac{C}{C_0} = \frac{Q/V}{Q/V_0} = \frac{V_0}{V} = \frac{E_0 \ell}{E \ell} = \frac{E_0}{E} = \frac{\sigma}{\epsilon_0} \frac{\epsilon_0}{\sigma - \sigma_i} =$$

$$K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma - \sigma_i = \frac{\sigma}{K}$$

$$E = \frac{\sigma - \sigma_i}{\epsilon_0} = \frac{\sigma}{K\epsilon_0}$$

$K\epsilon_0 = \epsilon$ = Permittivity of the dielectric

If $K = 1$, then ϵ_0 = permittivity of Vacuum

$$E = \frac{\sigma}{\epsilon}$$

Parallel Plate Capacitor, area A, separation l, dielectric K

$$C = KC_0 = \frac{K\epsilon_0 A}{l}$$

$$C = \frac{\epsilon A}{l}$$

For vacuum $K = 1$, $\epsilon \to \epsilon_0$

Energy $U = \frac{1}{2}CV^2$, $Q = CV$

$U = \frac{1}{2}\frac{Q^2}{C}$, w/o dielectric $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$

Put in dielectric $C = KC_0$

Charge unchanged $= Q_0$

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{KC_0} = \frac{U_0}{K}$$

Work to put in dielectric

$$W = U_0 - U = \frac{1}{2}C_0V_0^2\left(1 - \frac{1}{K}\right)$$

If $K = 1 \to W = 0$

Energy density in dielectric in Capacitor: $U = \frac{1}{2} C V^2$

$= \frac{1}{2} K C_0 V^2 = K U_0$

$= \frac{1}{2} K \epsilon_0 E^2 \cdot \text{Volume} = \frac{1}{2} \epsilon E^2 \text{ Vol}$

Difference is Voltage is held constant ⇒ supply remains connected and provides more energy to field when dielectric is put in → $U = K U_0$ ← supply adds charge

(disconnected: $U = \frac{1}{K} U_0$ → charge stays same)

Energy density in a E-field in dielectric

$U = \frac{1}{2} \epsilon E^2$

Electrons in a wire

random motion

Just as many move left to right as right to left

⇒ net flux thru plane = 0

Hook a power supply to ends of wire ⇒ Potential diff. b/w ends. ⇒ $\vec{E}$ is set up

electrons move in direction $-\vec{E}$

⇒ Current I established

If ΔQ passes thru any cross section of conductor in time Δt

$I = \frac{\Delta Q}{\Delta t}$, Units $\frac{\text{Coulombs}}{\text{second}}$

= Ampere

milliamps (10^{-3} A) mA

Microamps (10^{-6} A) μA