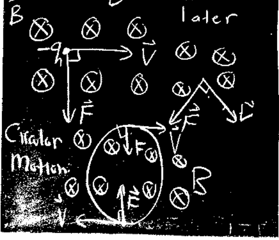
Test charge velocity $\vec{V}$ in magnetic field $\vec{B}$

Test charge = e^-



Charge stays in a plane goes around a circle

$F = -q_h V_\perp B = -q_h V B \sin\phi$

but $= 90° \Rightarrow F = -q_h V B$

Supplies centripetal force

$F = m a_\perp = m v^2 / r$

$\frac{m v^2}{r} = q_h V B \Rightarrow \boxed{r = \frac{mV}{q_h B}}$

m = mass of charged particle

angular velocity

$\omega = \frac{V}{r} = \frac{V}{\left(\frac{mV}{q_h B}\right)} = \frac{q_h B}{m}$

angular frequency →

$f_h = \frac{\omega}{2\pi} = \frac{q_h B}{2\pi m}$

frequency

Period $T = \frac{1}{f} = \frac{2\pi m}{q_h B}$

f, T do not depend on V
radius does ⇒ fast particles have larger radii for same B
Radius is proportional to particle momentum $m\vec{v} = \vec{p}$
$\vec{F} \perp \vec{v} \Rightarrow$ No Work done by $\vec{B}$

Charged particle in magnetic + electric fields:

Tune $\vec{E}, \vec{B}$ so forces Balance: $F_E = F_B$:

$eE = evB$

$E = vB$ matched forces

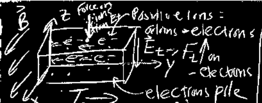
accelerate electrons thru potential difference V
energy $eV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2eV}{m}}$ Balance E, B forces $E = vB$ or $v = E/B$

$\sqrt{\frac{2eV}{m}} = \frac{E}{B} \Rightarrow \frac{2eV}{m} = \frac{E^2}{B^2}$

$\Rightarrow \frac{e}{m} = \frac{E^2}{2B^2V}$ ⇒ Thompson used to find the electron e/m ratio

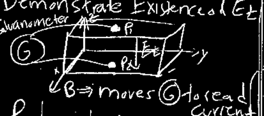
Conductor: electrons in a magnetic field:



Current along +y
$\vec{B}$ in $\vec{x}$ direction
electrons deflected down, until pile up at Bottom ⇒ excess of −charge at bottom, +charge (ions) at top

Positive charge at top
Negative charge at bottom
⇒ Transverse electric field $= E_t$ down
pushes up: $F_e = -eE_t$
Places downward force on positive ions connected to bar pushes down on holder (nothing holding → moves)

Demonstrate Existence of E_t



P_1 has higher potential than P_2 ⇒ charge carriers are electrons.

Hall Effect (1879)

Steady State (Forces Balance) $qE_t = qvB$

Current density: $J = n q_h V_d$

$V_d = \frac{J}{n q_h} \Rightarrow E_t = \left(\frac{1}{n q_h}\right) J B$

$\Rightarrow n q_h = \frac{J B}{E_t}$ ← measure

Hall Coefficient

How is magnetic Field Produced by a Current

Remember $\vec{E}$: charge dq_h

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{dq_h}{r^2}\right)\hat{r}$$

Now Look at wire + $\vec{B}$

Piece of wire $d\vec{s}$

I differential element of wire w/ Current I

$I\,d\vec{s}$ making angle θ w/ $\vec{r}$

Contribution to $\vec{B}$ $d\vec{B}$ from $I d\vec{s}$ is

$$dB = \frac{\mu_0}{4\pi} \frac{I\,ds \sin\theta}{r^2}$$

μ_0 = Constant = $4\pi \times 10^{-7}$ T·m/A

direction of $d\vec{B}$ is that of $d\vec{s} \times \hat{r}$

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{I\,d\vec{s} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s}\times\hat{r}}{r^2}$$

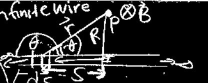
Biot-Savart Law

⇒ Total Contribution to $\vec{B}$

$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{s}\times\hat{r}}{r^2}$$

Integrate along wire)

Apply to long straight wire.



$$dB = \frac{\mu_0}{4\pi} I ds \frac{\sin\theta}{r^2}$$

$$B = \int_{-\infty}^{\infty} dB = \int_{-\infty}^{\infty} \frac{\sin\theta \, ds}{r^2}$$

$$\sin\theta = \sin(\pi-\theta) = \frac{R}{\sqrt{s^2+R^2}}$$

$$r = \sqrt{s^2+R^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{R}{(s^2+R^2)^{3/2}}\,ds$$

$$= \frac{\mu_0 I}{4\pi R}\left[\frac{s}{(s^2+R^2)^{1/2}}\right]_{-\infty}^{\infty}$$

$$= \frac{\mu_0 I}{4\pi R}(1 - -1) = \boxed{\frac{\mu_0 I}{2\pi R}}$$

Field around wire at distance R

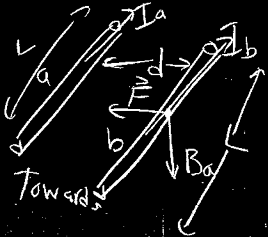
Direction → Right Hand Rule

Th

Co

→ Thumb along current I
Curl fingers around B

Force between 2 wires:



$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

$$F_b = B I L$$

$$= \frac{\mu_0 I_a I_b}{2\pi d} L$$

Current same direction → Towards
opposite → push away