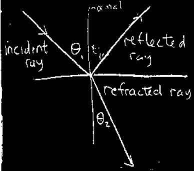
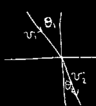
Phys 202
Exam 3 – Wed 4/20
5:45 - 6:45 pm

Refraction



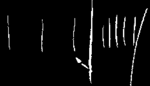


Angle of refraction satisfies

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

v = speed of light in medium

lower speed ⇒ ray closer to normal



Define index of refraction of material

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}}$$

$$n = \frac{c}{v}$$

$n \geq 1$

[frequency stays same
wavelength changes]

also note

Since $v_1 = f\lambda_1$ and $v_2 = f\lambda_2$

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$\Rightarrow \boxed{\lambda_1 n_1 = \lambda_2 n_2}$$

Since $\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$ and $\frac{v_1}{v_2} = \frac{n_2}{n_1}$,

$$\Rightarrow \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$ Snell's Law

Reflection: $\theta_i = \theta_r$

Refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

- Index of refraction depends on λ
  - dispersion
  - prism

- Total internal reflection

if θ_2 is large enough and $n_2 > n_1$, then cannot satisfy Snell's law $\Rightarrow$ only reflect

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

note: $\sin\theta_1 \le 1$

So. $n_2 \sin\theta_2 \le n_1$

$$\sin\theta_2 \le \frac{n_1}{n_2}$$

critical angle θ^* s.t.

$$\sin\theta^* = \frac{n_1}{n_2}$$

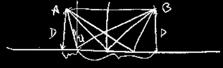
if $\theta_2 > \theta^*$, then

total internal reflection

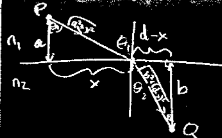
§35.9 Fermat's principle

Fermat 1601-1665

When light travels between any two points, its path is the one that requires the smallest amount of time.



reflection



Fermat

Speed of light in ① is c/n_1

② is c/n_2

$$\text{time in ①} = \frac{\sqrt{a^2+x^2}}{(c/n_1)}$$

$$\text{time in ②} = \frac{\sqrt{b^2+(d-x)^2}}{(c/n_2)}$$

$$\text{Total time} = \frac{n_1}{c}\sqrt{a^2+x^2} + \frac{n_2}{c}\sqrt{b^2+(d-x)^2}$$

$$\text{minimize: } \frac{d}{dx}\left[\frac{n_1}{c}\sqrt{a^2+x^2} + \frac{n_2}{c}\sqrt{b^2+(d-x)^2}\right] = 0$$

$$\frac{n_1}{c}\frac{x}{\sqrt{x^2+a^2}} + \frac{n_2}{c}\frac{-(d-x)}{\sqrt{b^2+(d-x)^2}} = 0$$

$$n_1 \frac{x}{\sqrt{x^2+a^2}} = \frac{n_2(d-x)}{\sqrt{b^2+(d-x)^2}}$$

$$\Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

§35.6 Huygen's principle

Huygens 1629–1695

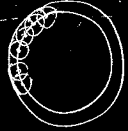
Derive laws of reflection and refraction (as well as diffraction) using geometric construction due to Huygens.

Huygens' principle: all points on a given wavefront are taken as point sources for spherical waves which propagate outward with speeds characteristic of waves in that medium

→ New position of the wavefront is the surface tangent to the wavelets.

1) plane wave

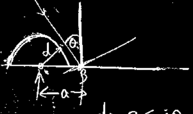
2) spherical wave:



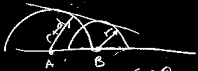
b) Reflection:

hit point A before point B

When wavefront hits B:



$d = a\sin\theta_i$



$(d+r) - a\sin\theta_r = r$

$a\sin\theta_i + r - a\sin\theta_r = r$

$\Rightarrow \sin\theta_i = \sin\theta_r$