Equations for image formation

$$\boxed{\frac{1}{q} + \frac{1}{p} = \frac{1}{f}}$$

p = distance to object
q = distance to image
f = focal length

$$\boxed{\frac{h'}{h} = -\frac{q}{p}}$$

h = height of object
h' = height of image

Concave mirror

$$f = R/2$$

magnification of concave mirror

$$\frac{1}{q} + \frac{1}{p} = \frac{2}{R} = \frac{1}{p}\left(1 + \frac{p}{q}\right) = \frac{2}{R}$$

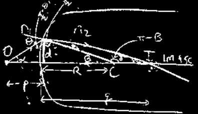
$$\frac{p}{q} = \frac{2p}{R} - 1$$

$$\Rightarrow \frac{h}{h'} = -\frac{p}{q} = 1 - \frac{2p}{R}$$

$$\frac{h'}{h} = \frac{1}{1 - 2p/R}$$

§36.3 Lenses [refraction]

1st 1 interface



$\tan\alpha = d/p$
$\tan\gamma = d/q$
$\tan\beta = d/R$

Use $\tan\theta \approx \theta$ for small θ

$\alpha \approx d/p$

$\beta \approx d/R$

$\gamma \approx d/q$

Use Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1\ \theta_2$ small $\Rightarrow \sin\theta \approx \theta$

$n_1\theta_1 = n_2\theta_2$

relate θ_1, θ_2 to α, β, γ.

$\perp\ \alpha + \beta + (\pi - \theta_1) = \pi \Rightarrow \theta_1 = \alpha + \beta$

$\theta_2 + \gamma + (\pi - \beta) = \pi$

$\Rightarrow \theta_2 = \beta - \gamma$

eliminate θ_1, θ_2

$n_1(\alpha+\beta) = n_2(\beta-\gamma)$

$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$

$n_1\frac{d}{p} + n_2\frac{d}{q} = (n_2-n_1)\frac{d}{R}$

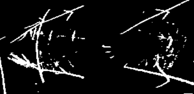
$$\Rightarrow \boxed{\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2-n_1}{R}}$$

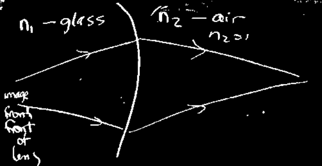
Sign convention

front side where light rays originate

real image formed by refraction is in back* of surface

Do rest of lens §36.4





done in book,

→ can write as

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

where focal length f given by

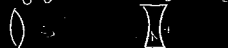
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

R_1 = radius of curvature of front of lens
R_2 = radius of curvature of back of lens

In limit of thin lens,

$$\boxed{\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

converging versus diverging lens



Magnification

Similar geometric construction as for mirror yields

$$M = h'/h = -q/p$$

Ray diagrams for thin lenses

converging lens

object farther than focal length:

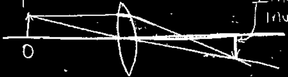


Image is real, inverted, + behind lens.

object closer than focal length

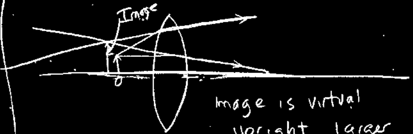


Image is virtual upright, larger than object on front side of lens

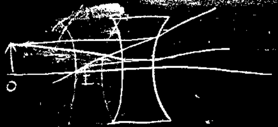
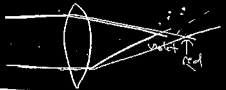


Image is virtual, upright, smaller than object in front of lens.

§36.5 Lens Aberrations

chromatic aberration:
different colors have different n.

typically



Spherical aberration:
rays at different positions on lens meet at same point only if close to principal axis

$(\tan\theta \approx \sin\theta \approx \theta)$