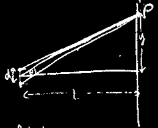
§37 Interference of light waves

Young's double-slit experiment

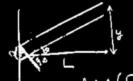
use Huygen's principle to study interference pattern

bright regions - constructive interference
dark regions - destructive interference



$d \ll L$

difference in path length



path length difference $= d\sin\theta$

get constructive interference if

$$d \sin\theta_{bright} = n\lambda$$

(n ↑ integer, λ ↑ wavelength)

destructive interference if

$$d\sin\theta_{dark} = (n+\tfrac{1}{2})\lambda$$

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if we also assume θ is small,

$\sin\theta \approx \tan\theta \; (\approx \theta)$

Since $y = L\tan\theta$ and $\sin\theta_{bright} = \frac{n\lambda}{d}$

$$y_{bright} = L\left(\frac{n\lambda}{d}\right) \quad n = 0, 1, 2, \ldots$$

$$y_{dark} = \frac{\lambda L}{d}\left(n+\tfrac{1}{2}\right) \quad n = 0, 1, 2, \ldots$$

path lengths are

$$\frac{L}{\cos\theta_1} \text{ and } \frac{L}{\cos\theta_2}$$

$$\tan\theta_1 = \frac{y - d/2}{L}$$
$$\tan\theta_2 = \frac{y + d/2}{L}$$

$\tan\theta = y/L$

$\cos\theta = \frac{L}{\sqrt{y^2+L^2}}$

$\sin\theta = \frac{y}{\sqrt{y^2+L^2}}$

$d \ll L$ so

θ_1, θ_2 are very close to θ

$$\tan\theta_1 = \tan\theta + \sec^2\theta\, \delta\theta = \frac{y - d/2}{L}$$

$$\frac{y}{L} + \frac{y^2+L^2}{L^2}\delta\theta = \frac{y - d/2}{L}$$

Solve for $\delta\theta$

$$\delta\theta = -\frac{L^2}{y^2+L^2}\left(\frac{d}{2L}\right)$$

path length difference is

$$L\sec\theta_2 - L\sec\theta_1$$
$$= L\left[\sec(\theta+\delta\theta) - \sec(\theta-\delta\theta)\right]$$

Use $\sec(\theta + \delta\theta) = \sec\theta + \delta\theta \sec\theta \tan\theta$

$$L\left[\sec\theta + \delta\theta \sec\theta\tan\theta - \left[\sec\theta - \delta\theta \sec\theta\tan\theta\right]\right]$$

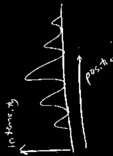
$$= 2L\,\delta\theta \sec\theta \tan\theta$$

$$= 2L\left(\frac{d}{2L}\right) \frac{L^2}{y^2+L^2} \frac{\sqrt{y^2+L^2}}{L} \frac{y}{L}$$

$$= d\,\frac{y}{\sqrt{y^2+L^2}}$$

$$= d\,\sin\theta$$

Intensity · double slit



Calculate intensity vs position.

$d\updownarrow$ add 2 waves with same amplitude + frequency but different phases.

$E_1 = E_0 \sin\omega t \qquad E_2 = E_0 \sin(\omega t + \varphi)$

phase difference proportional to path length difference:

if path length difference δ, then

$$\phi = \frac{2\pi\delta}{\lambda}$$

Find total E:

$$E = E_1 + E_2 = E_0\left(\sin\omega t + \sin(\omega t + \varphi)\right)$$

use $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$A = \omega t + \varphi,\ B = \omega t$

$$\Rightarrow E = 2E_0 \cos\varphi/2\ \sin(\omega t + \varphi/2)$$

intensity $= E^2 = 4E_0^2 \cos^2\varphi/2\ \sin^2(\omega t + \varphi/2)$

time average of intensity $\frac{1}{2}\left(4E_0^2\cos^2\varphi/2\right)$

$$= 2E_0^2\cos^2\varphi/2$$

$$= 2E_0^2\cos\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

if θ small,

$\sin\theta \approx \tan\theta \approx y/L$

$$I = I_{max}\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

§37.4 add waves using phasors

$E_1 = E_0 \sin \omega t \quad E_2 = E_0 \sin(\omega t + \varphi)$

choose t so that E_1 point along axis

$\tan\alpha = \dfrac{E_0 \sin\varphi}{E_0 + E_0\cos\varphi}$

$\tan\alpha = \dfrac{\sin\varphi}{1+\cos\varphi} = \dfrac{2\sin\varphi/2 \cos\varphi/2}{1 + (2\cos^2\varphi/2 - 1)} = \dfrac{\sin\varphi/2}{\cos\varphi/2} = \tan\varphi/2$

Magnitude:

$E_R^2 = (E_0 + E_0\cos\varphi)^2 + E_0^2 \sin^2\varphi$

$E_R^2 = E_0^2(1 + 2\cos\varphi + \cos^2\varphi + \sin^2\varphi)$

$= 2E_0^2(1+\cos\varphi)$

$= 2E_0^2(1 + 2\cos^2\varphi/2 - 1)$

$= 4E_0^2 \cos^2\varphi/2$

so $E_R = 2E_0 \cos\varphi/2$