The spin-fluctuation theory for the high $T_c$ superconductivity.

I. The normal state and the pairing.

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ICCMP Workshop, Nov. 19 2002
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Outline

- The cuprates
- Normal State and Quantum Criticality
- Quantum-Critical Pairing
- Conclusions
superconductor
underdoped
opt. doped
overdoped
antiferromagnet
pseudogap
superconductor
Facts:

- antiferromagnetism near half-filling
- $d$–wave symmetry of the superconducting state

Weak coupling theory yields $d$–wave spin-mediated pairing near AFM instability (Scalapino, Pines...)

Why there is still an interest in high $T_c$?

- non-Fermi liquid behavior in the normal state
- the pseudogap
To which extent superconductivity in the cuprates is the low energy phenomenon?

- The Fermi energy $E_F \sim 1eV$ ($p_F \sim 2.5/a$, $V_F \sim 0.8eV \times a$)

- The superconducting gap $\Delta \sim 40meV$

\[
\Delta/E_F \sim 0.04
\]

On the other hand, effective Hubbard-type interaction (responsible for antiferromagnetism) is $U \sim 1 - 2eV \sim E_F$.

Antiferromagnetism is the high-energy phenomenon!
Two different types of approaches to the cuprates

- doping of a quantum antiferromagnet (Mott insulator) (Sachdev, Lee, Anderson)
- approach antiferromagnetism from higher dopings
What happens when we approach antiferromagnetism from the paramagnetic side?

- is there a non FL behavior
- is there a superconductivity
- is there a pseudogap
- is there a secondary critical point at some distance from a magnetic QCP?
Quantum Critical Non-FL

Fermi Liquid

QCP
**SPIN-FERMION MODEL**

Describes the interaction between electrons and their *own* spin collective degrees of freedom

\[ 0 \xrightarrow{\Lambda} \text{Integrate Out} \xrightarrow{W} E \]

*Fermions* → *Static Spin Excitations*

- **Ingredients**
  - electrons near the Fermi surface
  - low-energy collective spin excitation
  - spin-fermion coupling (Hubbard $U$ in the RPA)

- **Input**
  - spin correlation length $\xi$
  - Fermi surface with hot spots
Spin decay into a particle-hole pair is allowed
\[ \mathcal{H} = \mathcal{H}_f + \mathcal{H}_{\text{spin}} + \mathcal{H}_{\text{int}} \]  

\[ \mathcal{H}_f = \sum_{\mathbf{k}, \alpha} v_F (\mathbf{k} - \mathbf{k}_F) c_{\mathbf{k}, \alpha}^\dagger c_{\mathbf{k}, \alpha} \]

\[ \mathcal{H}_{\text{spin}} = \sum_{\mathbf{q}} \chi_0^{-1}(\mathbf{q}) \vec{S}_q \vec{S}_{-\mathbf{q}} \]

\[ \mathcal{H}_{\text{int}} = g \sum_{\mathbf{q}, \mathbf{k}, \alpha, \beta} c_{\mathbf{k+q}, \alpha}^\dagger \vec{\sigma}_{\alpha, \beta} c_{\mathbf{k}, \beta} \cdot \vec{S}_{-\mathbf{q}} \]

The static spin susceptibility is an input

\[ \chi(q, \omega) = \frac{\chi_0}{\xi^{-2} + (Q - q)^2 - (\omega / v_F)^2}, \quad Q \approx (\pi, \pi) \]
The model has two typical energies

- \( \bar{\omega} \sim g^2 \chi_0 \) – effective interaction
  (at \( \omega > \bar{\omega}, \omega > \Sigma(\omega) \))

- \( \omega_{sf} \sim (v_F \xi^{-1})^2 / \bar{\omega} \propto \xi^{-2} \) – analog of the Debye frequency

The ratio of the two determines the dimensionless coupling constant

\[
\lambda = \left( \frac{\bar{\omega}}{4 \omega_{sf}} \right)^{1/2}
\]

- \( \omega_{sf} > \bar{\omega} \) – weak coupling
- \( \omega_{sf} < \bar{\omega} \) – strong coupling

\( \lambda \propto \xi \) diverges at the QCP.
Perturbation theory holds in $\lambda^{3-d}$

- logarithms in $d = 3$
- powers of $\lambda$ in $d = 2$

Perturbation theory does not work in $d = 2$ near the QCP.

Near optimal doping, $\omega_{sf} \sim 20 \text{meV}$,
$\bar{\omega} \sim 200 - 250 \text{meV}$, i.e. $\lambda \sim 1.5 - 2$

- for all relevant dopings, we are dealing with the strong coupling problem
- We have to solve simultaneously for fermionic and bosonic self-energies
Strong Coupling Results, Normal State

At $\lambda \geq 1$, spin fluctuations become soft compared to electrons due to a strong decay into particle-hole pairs, and Eliashberg theory becomes applicable.

- fermionic self-energy

\[ \Sigma(k, \omega) \approx \Sigma(\omega/\omega_{sf}) \]

- bosonic self-energy

\[ \chi^{-1} \propto 1 + (q - Q)^2 \xi^2 - i\omega/\omega_{sf} \]

Fermionic and spin excitations vary at the same scale $\omega_{sf}$. 
Fermi Liquid

Quantum Critical
Non-Fermi Liquid

\[ 0 \quad \omega_{\text{sf}} \sim \xi^{-2} \quad \omega \]

\[
\text{Im } \Sigma(\omega) \quad (\text{arb. units})
\]

\[ \omega \]

\[ \omega^2 \]

\[ 0 \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \]

\[ \omega / \omega_{\text{sf}} \]

\[
\chi^{1}(q,\omega) \sim q^2 + \xi^2
\]

\[
\chi^{1}(q,\omega) \sim q^2 + i \omega / \omega
\]

\[
G^{-1}(\omega) \sim \sqrt{\omega}
\]

Fermions: FL
Spin excitations: static

Fermions: QC NFL
Spin excitations: relaxational
The EDC photoemission intensity, optimal doping

Spectra at N

T=100K
FWHM=54 meV

A. Kaminski et al,
PRL 84, 1788 (2000)

Spectra at A

Bi2212 OP89K (100K)
The fermionic self-energy $\Sigma(\omega)$ extracted from the EDC photoemission data, optimal doping
The fermionic self-energy $\Sigma(\omega, T)$ extracted from the MDC photoemission data, optimal doping.
The energy dependence of the quasiparticle velocity $v_{eff} = v_F/(1 + \lambda(\omega))$ extracted from the MDC photoemission data, optimal doping
Real and imaginary part of optical conductivity at various temperatures, optimal doping
The imaginary part of the current-current polarization operator, optimal doping

\[ \sigma(\omega) = \frac{\omega^2_{pl}}{4\pi} \frac{i\Pi(\omega)}{\omega} \]

Theory

Experiment

\[ \text{Theory} \]

- T=100K
- T=200K
- T=295K

\[ \text{Experiment} \]

- \( \text{Im} \Pi(\omega) \)

Frequency (cm\(^{-1}\))

\[ \text{frequency (cm}^{-1}\text{)} \]

\[ 1000 \quad 2000 \quad 3000 \quad 4000 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \]

\[ \text{Im} \Pi(\omega) \]

\[ \text{frequency (cm}^{-1}\text{)} \]

\[ 1000 \quad 2000 \quad 3000 \quad 4000 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \]

\[ \text{Im} \Pi(\omega) \]
Quantum Critical Non-FL

Fermi Liquid
Pairing problem

- Spin-mediated pairing yields attraction in $d_{x^2-y^2}$ channel
  - Scalapino, Pines, Schrieffer ...

Which of the two scales, $\omega_{sf} \propto \xi^{-2}$ or $\bar{\omega} \sim 2J$ determines the pairing instability?
McMillan-type reasoning: \( T_c \sim \omega_{sf} \)

- at \( \omega < \omega_{sf} \), \( \lambda \) is reduced by a mass renormalization \( \Rightarrow \lambda_{eff} = \frac{\lambda}{1+\lambda} = O(1) \)
- pairing interaction decreases above \( \omega_{sf} \)

At \( \omega < \omega_{sf} \), the system behaves as a Fermi liquid with an effective pairing coupling \( O(1) \).

\[
T_c \sim \omega_D \exp \left[ -\frac{1+\lambda}{\lambda} \right]
\]
Alternative reasoning: $T_c \sim \bar{\omega}$

- above $\omega_{sf}$, $\lambda$ becomes $\lambda(\omega)$
- the reduction of $\lambda(\omega)$ reduces the mass renormalization
- $\lambda_{eff} \approx 1$ up to $\omega \sim \bar{\omega}$

A novel, universal, non BCS pairing problem: NFL fermions with attraction due to an exchange of gapless spin collective modes.
Numerical and analytical analysis:
A linearized $d-$wave gap equation has a solution at $T_{ins} \sim \bar{\omega}$
Consider $\omega_{sf} = 0$.

The linearized equation for the $d$-wave pairing vertex is

$$
\Phi_k(\omega) = \frac{\pi T}{2} \varepsilon_k \sum_{\omega'} \frac{\Phi_k(\omega')}{\sqrt{|\omega'| |\omega' - \omega|}} \frac{1}{1 + \sqrt{|\omega'|/\bar{\omega}}} + \Phi_0
$$

$\varepsilon_k = 1$ at a hot spot, and decreases away from a hot spot.

The kernel is $O \left( \frac{1}{\omega} \right)$ as in BCS theory, but now

- $|\omega|^{-1/2}$ comes from fermionic self-energy
- $|\omega|^{-1/2}$ comes from gapless collective mode

If a solution exists at $\Phi_0 = 0$, $T_{ins} \sim \bar{\omega}$, but no guarantee that this solution does exist.
Suppose $\epsilon_k \ll 1$ (far away from a hot spot)

We can perturbatively sum logarithms, as in BCS theory.

In BCS theory

$$\chi_{pp}(T) = \frac{\Phi(T)}{\Phi_0} = \frac{1}{\log T/T_c}$$

- the susceptibility diverges at $T_c$

- below $T_c$, $\chi_{pp}$ is negative

In our case

$$\chi_{pp} = \left( \frac{\overline{\omega}}{\omega} \right)^{\epsilon_k}, \quad T = 0$$

- the susceptibility remains positive, no instability
Near a hot spot, $\epsilon_k \approx 1$

- threshold at $\epsilon_k = \epsilon_{th} = 0.221$
- instability at $\epsilon_k > \epsilon_{th}$.

$$T_c \sim e^{-A/(\epsilon-\epsilon_{th})}, \quad A \approx 3$$

- the non BCS solution exists only in a finite range of $k$

At large $\lambda$, $\Delta(k)$ becomes peaked at hot spots
QCP

strong coupling

weak coupling

QC pairing

FL pairing

FL

QC NFL

T

doping x
CONCLUSIONS

• strong interaction between fermions and their own low-energy spin collective modes yields:
  
  - non-Fermi liquid, QC behavior in the normal state between
    \( \omega_{sf} \propto \xi^{-2} \) and noncritical \( \bar{\omega} \)
  
  - NFL behavior is consistent with photoemission and conductivity measurements
  
  - near the QCP, the system undergoes a non BCS pairing instability at \( T_{ins} \sim \bar{\omega} \)