New determination of the asymptotic D-state to S-state ratio of the triton using (d, t) reactions at sub-Coulomb energies

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The asymptotic D-state to S-state ratio of the $^3$H → n + d cluster wave function ($\eta_t$) has been determined from measurements of sub-Coulomb (d, t) tensor analyzing powers. New measurements of the tensor analyzing power $T_{20}$ for (d, t) reactions on $^{208}$Pb and $^{119}$Sn at several different energies are presented. These data, along with previous measurements of $T_{20}$ and $T_{21}$ for several additional reactions and energies, are used in a distorted-wave Born approximation analysis that includes nuclear tensor potentials as well as long-range tensor potentials that arise from Coulomb interactions. A weighted mean of all 14 statistically independent values gives the result $\eta_t = -0.0431 \pm 0.0025$, where the quoted uncertainty includes statistical and systematic contributions. This result is compared with previous measurements and with predictions obtained from Faddeev calculations.

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I. INTRODUCTION

At the present time, there is a great deal of interest in understanding the spin structure of light nuclei, particularly of the A = 3 nuclei, $^3$H and $^3$He. These nuclei represent an important testing ground for models of the NN interaction and for studies of the many-body aspects of the strong interaction in nuclei. One reason for this importance is that in the 3N system it is possible to carry out quantum-mechanical calculations that are essentially exact. In addition, this is the simplest system in which the effects of possible three-body nuclear forces can be explored. Also, we note that there are a number of experiments currently being planned (for example, experiments to study the scattering of electrons from the polarized neutron within a polarized $^3$He nucleus) that rely on a detailed knowledge of the spin structure of the bound-state wave functions of A = 3 nuclei.

The new work reported in this paper concerns the properties of the D-state component of the $^3$H wave function. The existence of this D-state component, which arises fundamentally from the NN tensor force, has been understood theoretically for 50 years. The D-state wave function is complex in form, with several distinct configurations allowed by parity and angular momentum conservation. Modern Faddeev calculations employing realistic NN interactions predict D-state probabilities in the neighborhood of 9%.

Unfortunately, direct experimental information concerning the nature of the D-state components of the A = 3 nuclei is relatively scarce, and what information there is is generally not of high accuracy. The only quantity directly associated with the $^3$H D state that can be reliably determined at the present time is the asymptotic D-state to S-state ratio of the $^3$H→ n + d cluster wave function, $\eta_t$. Measurements of this quantity are clearly of importance since they provide a means for testing the theoretical wave functions obtained by direct solution of the Faddeev equations. In addition to the general interest in obtaining experimental results that relate to the spin structure of $^3$H, there has recently been renewed interest in determinations of $\eta_t$, stimulated in part by theoretical work that has provided a clear prediction for this quantity. Until recently, however, $\eta_t$ had been determined experimentally to only 10–20%.

In this paper, we describe recent experimental work that has led to a new determination of $\eta_t$. This new determination is obtained by comparing measurements of the tensor analyzing powers for sub-Coulomb (d, t) reactions with distorted-wave Born approximation (DWBA) calculations of those analyzing powers. Our data set includes previously reported [1, 2] measurements of the tensor analyzing powers $T_{20}$ and $T_{21}$ for five different (d, t) reactions, as well as new measurements of $T_{20}$ for several reactions at lower energy and with higher precision than the old data. Our determination is based on a data set that is more extensive and includes measurements at lower energy (and is therefore less sensitive to uncertainties in the calculations) than for any previous determination of $\eta_t$ by this method.

In Sec. II, we discuss the background information which is relevant to this determination of $\eta_t$. Section III contains a description of the experimental details for the new analyzing power measurements. The DWBA analysis of the measurements is discussed in Sec. IV. Results are presented in Sec. V, and are discussed and compared to previous theoretical and experimental determinations in Sec. VI.

II. BACKGROUND

A. Previous determinations of $\eta_t$

At this time, there are clear theoretical predictions for $\eta_t$ [3–5]. These predictions are based on the observation
that if one does many different Faddeev calculations of triton observables (using different two- and three-nucleon potentials and different numbers of channels), there is a linear correlation between \( \eta_t \) and the triton binding energy \( E_t \) [3, 4], and also between \( \eta_t/\eta_d \) and \( E_t \) [5]. If this linear correlation is evaluated at the experimentally determined value of the binding energy, \( E_t = -8.48 \text{ MeV} \), a prediction for \( \eta_t \) is obtained. The results from Refs. [3–5], which are listed in Table I, are reasonably consistent, differing from the average value, \( \eta_t = -0.044 \), by no more than 5%. Because these predictions are based on general principles and are so consistent, experimental tests of them are of immediate interest.

Previous experimental determinations of \( \eta_t \) fall into one of two categories: those obtained from extrapolation in angle of \( ^2\mathrm{H}(\vec{d}, p)^3\mathrm{H} \) or \( ^4\mathrm{He}(\vec{d}, {^{3}\mathrm{He}})^3\mathrm{H} \) spin observables to the neutron exchange pole (for example, Refs. [6–8]), and those obtained from distorted-wave Born approximation (DWBA) analysis of tensor analyzing powers for \( (\vec{d}, t) \) reactions, usually on heavy nuclei (for example, Refs. [1, 2, 9–11]). The first method (pole extrapolation) relies on the fact that tensor analyzing powers for reactions that involve the \( ^3\mathrm{H} \to n + d \) vertex depend in a simple way on the triton asymptotic normalization constants at the neutron exchange pole. For physical values of the incident deuteron energy, the neutron exchange pole is at unphysical angles. Therefore, what is done is to fit the measured observables in the physical region with some function and use that function to extrapolate to the pole. One of the most difficult and controversial aspects of this method is the treatment of the systematic errors in the extrapolation procedure (see Refs. [8, 12, 13]).

Values for \( \eta_t \) obtained by this pole extrapolation method are given in Table I. The authors claim that the quoted errors include the effect of uncertainties in the extrapolation procedure, although this claim has not been independently verified. These results for \( \eta_t \) are consistent with the theoretical predictions at the one-standard-deviation level, but the uncertainties in these experimental determinations are somewhat large.

The second method for determining \( \eta_t \) is based on the comparison of tensor analyzing powers for \( (\vec{d}, t) \) reactions to DWBA calculations. In early experiments of this type, the DWBA calculations were carried out using the local energy approximation (LEA) [14]. In the LEA, the \( D \)-state parameter that is determined is \( D_2 \), which is approximately related to \( \eta_t \) by \( D_2 \approx \eta_t/\alpha^2 \), where

\[
\alpha^2 = \frac{2m_d m_n (E_t - E_d)}{(m_d + m_n)^2}.
\]

Karban and Tostevin [9] analyzed tensor analyzing power measurements for five different \( (\vec{d}, t) \) reactions on heavy nuclei and obtained \( D_2 = -0.22 \pm 0.02 \) (corresponding to \( \eta_t = -0.044 \pm 0.004 \)). This was the first such analysis to include tensor potentials in the deuteron channel. More recent determinations are based on full finite-range DWBA calculations. These experiments determine \( \eta_t \) directly. For example, Bhat et al. [10] obtained a value for \( \eta_t \) of \(-0.050 \pm 0.010 \) by comparing full finite-range DWBA calculations including deuteron tensor potentials with analyzing powers measured for \( ^{31}\mathrm{P}(\vec{d}, t)^{30}\mathrm{P} \) at 16 MeV (see Table I). Uncertainties in the \( l \)-mixing amplitudes and in the optical model parameters used in the calculations are significant sources of uncertainty in this result.

If deuteron energies below the Coulomb barrier are used, the sensitivity to the optical model potentials is greatly reduced. Also, at low energy the tensor analyzing powers are, to a very good approximation, proportional to \( \eta_t \). The calculated tensor analyzing powers are still somewhat dependent on the optical model potentials used in the calculation, especially on the deuteron-nucleus tensor potentials. It is important to estimate this contribution to the uncertainty carefully, since it can be significant, especially at energies not far below the Coulomb barrier.

The most recent value for \( \eta_t \) obtained from the DWBA analysis of sub-Coulomb tensor analyzing power data [11] is given in Table I. The tensor analyzing power data used for this determination were from three different \( (\vec{d}, t) \) reactions at deuteron energies ranging from 85% to 115% of the Coulomb barrier. (We take the Coulomb barrier to be the maximum height of the Coulomb potential plus the real part of the nuclear central potential, using the deuteron-nucleus optical model potential of Daehnick [15].) We believe [16] that the uncertainty in the result reported in Ref. [11] has been underestimated by a factor of 2 due to an underestimation of the error arising from the optical model potentials. The authors of Ref. [11] neglect the error arising from the uncertainty in the nuclear tensor potentials, whereas we find this to be the dominant error contribution. Even with the increased error, however, this determination of \( \eta_t \) is more precise than any previous determination and is consistent with the theoretical predictions.

### TABLE I. Theoretical and experimental determinations of \( \eta_t \).

<table>
<thead>
<tr>
<th>Method</th>
<th>( \eta_t )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theory</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation of ( \eta_t ) and ( E_t )</td>
<td>-0.0432(15)</td>
<td>[3]</td>
</tr>
<tr>
<td>Correlation of ( \eta_t ) and ( E_t )</td>
<td>-0.046(1)</td>
<td>[4]</td>
</tr>
<tr>
<td>Correlation of ( \eta_t/\eta_d ) and ( E_t )</td>
<td>-0.0430(12)*</td>
<td>[5]</td>
</tr>
<tr>
<td><strong>Experiment</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pole extrapolation, ( ^2\mathrm{H}(\vec{d}, p)^3\mathrm{H} )</td>
<td>-0.048(7)</td>
<td>[6]</td>
</tr>
<tr>
<td>Pole extrapolation, ( ^2\mathrm{H}(\vec{d}, p)^3\mathrm{H} )</td>
<td>-0.051(5)</td>
<td>[7]</td>
</tr>
<tr>
<td>Pole extrapolation, ( ^4\mathrm{He}(\vec{d}, ^3\mathrm{He})^3\mathrm{H} )</td>
<td>-0.050(6)</td>
<td>[8]</td>
</tr>
<tr>
<td>LEA DWBA analysis</td>
<td>-0.044(4)</td>
<td>[9]</td>
</tr>
<tr>
<td>DWBA analysis, ( ^{31}\mathrm{P}(\vec{d}, t)^{30}\mathrm{P} )</td>
<td>-0.050(10)</td>
<td>[10]</td>
</tr>
<tr>
<td>Sub-Coulomb DWBA analysis</td>
<td>-0.043(4)b</td>
<td>[11]</td>
</tr>
<tr>
<td>This experiment</td>
<td>-0.0431(25)</td>
<td>—</td>
</tr>
</tbody>
</table>

* Using \( \eta_d = 0.0256(4) \) (Ref. [29]).
  
  b Uncertainty from Ref. [16].

B. Present determination

We have made a new determination of \( \eta_t \) using an extensive set of \( (\vec{d}, t) \) tensor analyzing power measurements.
Some of the measurements we use [1, 2] were obtained at our laboratory about ten years ago for use as part of a similar, but less sophisticated, study of the triton D state. We have also obtained new data, making use of the increased beam current and fast polarization switching capability of our ion source [17] and an improved calibration of our deuteron polarimeter [18] to obtain more precise data, and data at lower energy, than was possible ten years ago. The deuteron energy of the data used in this determination of η₉ ranges from 69% to 95% of the height of the Coulomb barrier. The lower energy (compared to Ref. [11]), plus the fact that our data set is more extensive (nine different energy, target, and final state combinations, compared to four for Ref. [11]), means that the present determination of η₉ represents a significant improvement over previous work.

Our analysis is based on a full finite-range DWBA calculation, including nuclear tensor potentials. We also include the effects of long-range tensor potentials due to the Coulomb interaction of the deuteron and target nucleus. A significant part of the analysis is devoted to the estimation of the uncertainty in the result, especially the calculational uncertainties.

III. DESCRIPTION OF THE EXPERIMENT

We have measured the tensor analyzing power T₂₀ [19] of the ¹¹⁹Sn(ⁿ, t)¹¹⁸Sn reaction at 6.0 and 4.9 MeV, and of the ²⁰⁸Pb(ⁿ, t)²⁰⁷Pb(Eₐ = 0.00, 0.57, 0.90 MeV) reactions at 9.0 MeV. We have also measured T₂₀ for the ¹¹⁹Sn(ⁿ, t)¹¹⁸Sn reaction at higher energies (7.0, 8.0, and 9.0 MeV) in order to determine the energy dependence of the tensor analyzing powers and to test the method of determining the uncertainty in the DWBA calculations.

The measurements were made at the University of Wisconsin tandem accelerator laboratory, using the deuteron beam from the crossed-beam polarized ion source [17]. The spin alignment axis of the beam was along the beam momentum direction so that the t₂₀ beam moment was maximized, while the other components of the beam polarization were kept close to zero. The sign of the beam polarization was cycled between positive, negative, and unpolarized states at intervals of less than 1 sec. A polarimeter [18] located at the exit of the main scattering chamber was used to monitor continuously all relevant components of the beam polarization.

Beam-defining slits 1 mm high by 1.5 mm wide were placed at the entrance to the chamber. A tantalum anticorrelation slit 3.2 mm in diameter was used to intercept deuterons that scattered off the edges of the beam-defining slits. The targets were enriched self-supporting foils. The ²⁰⁸Pb target was isotopically enriched to 99.86% and was 1.01 mg/cm² thick. The ¹¹⁹Sn targets were enriched to 85.0%; the target thickness was 1.95 mg/cm² for the 4.9 MeV measurements, and 2.62 mg/cm² for the other measurements. Reaction products were detected in three ΔE-E telescopes placed on the beam left side of the main scattering chamber, 10 cm from the target. The telescopes consisted of rectangular silicon surface-barrier detectors. The ΔE detectors were 60 μm thick for the 4.9, 6.0, and 7.0 MeV ¹¹⁹Sn measurements and 100 μm thick for the rest of the measurements. The E detectors were 2500–3000 μm thick. Rectangular slits 15.9 mm high by 3.2 mm wide defined detector solid angles of 4.9 mrs.

Signals from the ΔE and E detectors of each telescope were processed separately, and then fed into two ADCs which were operated in coincidence mode. The ΔE and E signals were stored in a two-dimensional histogram, so that the difference in energy loss in the ΔE detector could be used to identify protons, deuterons, and tritons in the spectrum.

A sample ΔE-E spectrum for the ²⁰⁸Pb(ⁿ, t)²⁰⁷Pb reaction is shown in Fig. 1(a). The three peaks seen in this spectrum correspond to tritons from reactions populating the ²⁰⁷Pb final states at Eₐ = 0.00, 0.57, and 0.90 MeV. The counts in the lower left corner of this spec-

![Image](a)

**FIG. 1.** Sample spectrum for the ²⁰⁸Pb(ⁿ, t)²⁰⁷Pb reaction at Eₐ = 9.0 MeV and a laboratory angle of 160°. In (a), a two-dimensional ΔE-E spectrum is shown. The dashed lines show the limits used in producing the one-dimensional spectrum for the transition to the second excited state of ²⁰⁷Pb. In (b) is shown the one-dimensional spectrum obtained from the two-dimensional spectrum by the method described in the text. This spectrum represents about 2 h of running time.
trum are from the low-energy tail of the $d^{208}$Pb elastic scattering peak and from deuteron elastic scattering from an $^{16}$O contaminant in the target. We extract the triton peak sum by first projecting the region containing the triton peak onto a line perpendicular to the triton locus. The dashed line in Fig. 1(a) shows the region to be projected for the second excited state of $^{207}$Pb. This results in a one-dimensional spectrum, as shown in Fig. 1(b). The background in the region of the triton peak is mostly due to pileup in the detectors and to the tail of the deuteron locus. The analyzing power of the background was statistically zero. The peak-to-background ratio was greater than 100 to 1 in all cases, except for the 4.9 MeV $^{119}$Sn($d$, $t^{118}$Sn measurements (Fig. 2). In this case, the background was about 15% of the triton peak sum.

For the spectra in which the background was small, the contribution of the background to the peak was subtracted by determining the number of background counts in representative regions on both sides of the triton peak in the one-dimensional spectrum. The background was assumed to be linear. For each spectrum, several different choices of background regions were used so the uncertainty due to the choice of cuts could be estimated. The subtracted background was less than 2% of the total peak sum in all these cases, and the resulting fractional change in $T_{20}$ was also less than 2%.

Because the background was so large for the 4.9 MeV $^{119}$Sn data, the background subtraction was less straightforward. Peak fitting suggests that the deuteron peak does not extend under the relatively narrow window used for obtaining the triton peak sums. Therefore, the background was subtracted away by assuming that the background was linear, and that the region above the triton peak could be used to characterize the background. The region above the peak was fit with a line, which was then extrapolated back under the peak sum region to determine the background to be subtracted. The background correction obtained in this way ranged from 7% to 17% of the measured analyzing powers.

To obtain an estimate of the uncertainty in this correction, we subtracted away the background in a different way, by assuming a linear background and using regions on both sides of the peak to determine the background, just as was done for the rest of the data. As can be seen in Fig. 2(b), this method leads to larger background corrections. We believe the first method of background subtraction is probably more reasonable. The difference between the corrections obtained using the two different methods, however, gives a good estimate of the range of possibilities for the background correction. The uncertainty estimated in this way ranged from 2% to 7% of the measured analyzing powers, or about 40% of the correction itself. Because the background correction depends strongly on the method of background subtraction, and does so in the same way for each angle, the uncertainty in the background correction is correlated from angle to angle.

The analyzing power $T_{20}$ was determined by using the expression

$$T_{20} = \frac{R - 1}{t_{20}^+ - R t_{20}^-},$$

where $R = F^+ / F^-$. The quantities $F^+$ and $F^-$ represent the number of counts divided by the integrated charge in the state with beam polarization $t_{20}^+$ and the state with beam polarization $t_{20}^-$, respectively. We used detectors on the left side of the beam only, so there is a small correction to $T_{20}$ due to the small but nonzero $t_{21}$ and $t_{22}$ beam moments. The data analysis was initially done by assuming that the non-$t_{20}$ beam moments are identically zero. A DWBA calculation that fits the $T_{20}$ data was then used to determine values for $T_{21}$ and $T_{22}$, and these calculated values were used with the measured beam moments to determine a correction to the $T_{20}$ data. In all cases, this correction is less than 2% of

![Diagram](image_url)

Fig. 2. Sample spectrum for the $^{119}$Sn($d$, $t^{118}$Sn reaction at $E_d = 4.9$ MeV and a laboratory angle of 156°. In (a), a two-dimensional spectrum is shown. The dashed lines show the limits used in producing the one-dimensional spectrum. In (b) the one-dimensional spectrum is shown. The lines under the triton peak show the two methods of determining the background; the long dashes correspond to the linear extrapolation of the high-channel background only, while the short dashes correspond to a linear fit to the high- and low-channel background. This spectrum represents about 100 h of running time.
the measured analyzing powers.

The measured $T_{20}$ analyzing powers are shown in Fig. 3, Fig. 4, and Fig. 5. The uncertainties shown in the figures include statistical errors in the peak sums, background sums, and beam moments; uncertainties in the correction for nonzero $t_{21}$ and $t_{22}$ beam moments; and uncertainties arising from the choice of the background and peak sum regions. In addition to the corrections for background and for nonzero off-axis beam moments, corrections were made for electronic dead time and for errors in integrated charge due to finite response time of the integrating circuit. All corrections were less than 2% of the measured analyzing powers, except the background correction for the $^{119}$Sn data, which ranged from 7% to 17% of the measured analyzing powers, as mentioned before. The error bars shown in Fig. 4 for the 4.9 MeV $^{119}$Sn analyzing powers do not include the uncertainty in the background correction. This is because the background correction at this energy is dominated by the systematic errors associated with the choice of background subtraction method, which means that the background errors are correlated from angle to angle. We will return to the effect of the background uncertainty in Sec. IV D, where

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3}
\caption{Comparison of DWBA calculations with measured tensor analyzing powers $T_{20}$ for $^{208}$Pb($d,t)^{207}$Pb at $E_d = 9.0$ MeV. The error bars include the statistical uncertainty and the uncertainty in the corrections for background and nonzero off-axis beam moments. The solid line is a DWBA calculation with $\eta_t = -0.043$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig4}
\caption{Comparison of DWBA calculations with measured tensor analyzing powers $T_{20}$ for $^{119}$Sn($d,t)^{118}$Sn at $E_d = 6.0$ and 4.9 MeV. The previously measured data at 6.0 MeV are shown as open circles. The comments of Fig. 3 apply.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig5}
\caption{Comparison of DWBA calculations with measured tensor analyzing powers $T_{20}$ for $^{119}$Sn($d,t)^{118}$Sn at $E_d = 7.0, 8.0,$ and $9.0$ MeV. The comments of Fig. 3 apply.}
\end{figure}
we discuss the determination of the uncertainty in $\eta_r$. The old measurements of $T_{20}$ for the $^{116}\text{Sn}(d, t)^{118}\text{Sn}$ reaction at 6.0 MeV are shown along with the new measurements in Fig. 4. The old $^{116}\text{Sn} T_{20}$ data have been corrected for the recalibration of the polarimeter that was recently performed by Rodning [20]. These corrections were less than 2%.

IV. THE DWBA ANALYSIS

A. General considerations

For several reasons, DWBA calculations are expected to be very reliable for $(d, t)$ reactions at sub-Coulomb energies [21]. Since the Coulomb barrier excludes the incident and outgoing particles from the region near the target nucleus, the nuclear potential has little effect on the reaction. Also, the incoming and outgoing distorted waves are nearly pure Coulomb waves, and are therefore known accurately.

The choice of bombarding energy has an important effect on the determination of $\eta_r$. At lower bombarding energies, the DWBA calculations are less sensitive to the nuclear potential, which means that the systematic errors from uncertainties in the calculations are smaller. This is important, since a large part of the uncertainty in the result comes from the uncertainties in the scattering and bound-state wave functions. On the other hand, the reaction cross section drops rapidly with energy; therefore, the experiments become more difficult and the statistical errors are larger at lower energies.

The specific reactions studied were chosen to reduce calculational uncertainties. Reactions with Q values near zero are advantageous, since in this case the incoming and outgoing scattering wave peak at nearly the same radial distance, enhancing the overlap and therefore reducing the contribution to the reaction amplitude from the region near the nucleus. If the angular momentum transferred by the neutron ($j_n$) is unique, there is no uncertainty in the calculation of the analyzing powers due to uncertainties in spectroscopic factors. All of the reactions in our data set have unique $j_n$. Most of the reactions have $j_n = l_n + 1/2$, since tensor analyzing powers for these transitions tend to be larger in magnitude than tensor analyzing powers for transitions with $j_n = l_n - 1/2$ (see Ref. [22]). In addition, the transitions chosen have fairly large spectroscopic factors to reduce the contributions to the reaction from multistep and collective processes. The target nuclei used have no low-lying collective states.

B. Details of the calculation

The DWBA calculations were performed using a version of the full finite-range program PTOLEMY [23] that has been modified to include spin degrees of freedom and to permit the use of tensor deuteron-nucleus potentials. This version of PTOLEMY has also been modified to be suitable for light-ion transfer reactions.

The central and spin-orbit parts of the optical potential in the deuteron channel were taken from Daehnick et al. [15]. The optical potential in the triton channel was taken from Becchetti and Greenlees [24]. A measure of the sensitivity of the calculations to the choice of optical model potentials can be obtained by setting each of the potential depths to zero individually. For the 10 MeV $^{208}\text{Pb}(d, t)^{207}\text{Pb}$ reactions (the reactions in our set most sensitive to the nuclear potentials) it was found that the real central and spin-orbit components do not have a significant effect (less than 2%) on the calculations. Turning off the absorptive potentials, however, produced a 10% or larger effect on the calculated tensor analyzing powers. Thus, except for the absorptive potentials, the particular central and spin-orbit potentials chosen have little effect on the calculated analyzing powers.

In addition to the central and spin-orbit terms, the potential also includes a nuclear tensor potential of the form

$$[V_{TR}(r) + i W_{TR}(r)] T_r,$$

where $T_r = (s \cdot r)^2 - \frac{3}{4}$. The choice of tensor potentials can affect the calculated tensor analyzing powers significantly at the energies represented in our data set. The folding model predicts both real and imaginary parts to the tensor potential [25]. However, tensor analyzing power measurements for $^{208}\text{Pb}(d, d)$ at 10.0 MeV [26], 9.0 MeV [27], and 8.0 MeV [28], for $^{90}\text{Zr}(d, d)$ at 5.5 MeV [27], and for $^{136}\text{Xe}(d, d)$ at 5.5 MeV [29] are not well reproduced by the folding model values. When used in conjunction with conventional central potentials, the full folding model tensor potential produces analyzing powers that are much larger in magnitude than the measurements. Reasonable agreement with the data can be achieved, however, by omitting either the real or the imaginary part of the potential, or by dividing both the real and the imaginary parts by a factor of 2 [except for the $^{90}\text{Zr}(d, d)$ data, for which these calculated analyzing powers are still too large in magnitude]. In general, for any value of the real tensor potential between zero and the folding model value, a value for the imaginary tensor potential can be found that allows the elastic scattering data to be reproduced. We have chosen to use half the folding model strengths for the real and imaginary parts of the tensor potential for all reactions and energies in the data set. The effects of the uncertainty in the tensor potential are discussed in Sec. IV D.

In addition to the nuclear tensor potentials, we include two long-range tensor potentials that arise from Coulomb interactions between the deuteron and the target nucleus. The first,

$$V_{QT} = \frac{3}{2} Q Z e^2 r^{-3} T_r,$$

arises from the interaction of the deuteron quadrupole moment with the electric field gradient of the target nucleus. The strength and form of this potential are well known, since the interaction is purely electromagnetic and depends on the size of the deuteron quadrupole moment. The second long-range potential,

$$V_p = -\frac{1}{2} z^2 e^2 r^{-4} (\alpha + 3T_r),$$


is due to the electric polarization of the deuteron in the Coulomb field of the nucleus. This potential has a central part, which is proportional to \( \alpha \), and a tensor part, which is proportional to \( \tau \). The tensor part arises from the fact that the deuteron is more easily polarized when the electric field is along its spin alignment axis. The central term has a negligible effect on the analyzing powers, so we use only the tensor part of this potential in our calculation. We use the value \( \tau = 0.0343 \text{ fm}^3 \), calculated by Lopes et al. [30]. At the energies represented in our data set, including these two long-range potentials changes the calculated analyzing powers by a few percent of the analyzing powers.

The Coulomb field of the target nucleus produces electric polarization of the triton as well as the deuteron. These distortions can result in virtual \( P \)-state admixtures in the \(^3\text{H}\to n + d\) cluster wave function. The effect of these \( P \)-state admixtures on the tensor analyzing powers has not been calculated, so we are not able to include these corrections in our results. For \((d,t)\) reactions, the contribution of virtual \( P \) states in the deuteron wave function to the tensor analyzing powers is approximately 2% [31]. One would expect the effect of \( P \)-state admixtures in the triton wave function on \((d,t)\) reactions to be somewhat less than this, because the triton is more tightly bound than the deuteron and also because the effect of the deuteron wave function on the analyzing powers is small [21].

The triton bound-state wave function used in the calculations was generated using the separation energy procedure. The potential used had a Woods-Saxon shape, with a radius parameter of \( r = 1.5 \text{ fm} \) (corresponding to a radius of 1.89 fm) and \( a = 0.5 \text{ fm} \) [10]; the potential depth was adjusted to reproduce the triton binding energy. The normalization of the \( S \) - and \( D \)-state components of the wave function was chosen to give \( \eta_t = -0.04494 \). We have chosen to use this wave function rather than a realistic one because its simple parametrization makes it easier to investigate the effect of uncertainties in the wave function on the calculations. We will discuss the adequacy of this choice of wave function in Sec. IV.D.

The bound-state wave function of the picked-up neutron was generated in the same way as the triton wave function, with a real central potential of Woods-Saxon form with \( r = 1.2 \text{ fm} \), \( a = 0.7 \text{ fm} \). A real spin-orbit potential with \( V_{SO} = 6.0 \text{ MeV} \) was also included.

### C. Extracting a value for \( \eta_t \)

To extract a value of \( \eta_t \) from our analyzing power measurements, we use the fact that the calculated tensor analyzing powers are very nearly proportional to the value of \( \eta_t \) used in the DWBA calculation. Thus, we can find the value of \( \eta_t \) that gives the best fit to the data by minimizing

\[
\chi^2 = \sum_i \left[ \frac{(\eta_t/\eta_0^t)\sigma_i - y_i}{\sigma_i} \right]^2.
\]

Here, \( y_i \) are the measured analyzing powers with statistical uncertainties of \( \sigma_i \). \( \sigma_i \) are the analyzing powers calculated using a triton wave function with \( \eta_0^t \), which in our case was -0.04494.

We have performed this minimization for all of the sub-Coulomb data. Results are listed in Table II, and are also represented graphically in Fig. 6. There are 14 statistically independent values of \( \eta_t \). Values of the \( \chi^2 \) per degree of freedom (\( \chi^2/d \)) and confidence levels are also given in the table for each value. Generally speaking, the \( \chi^2 \) values are acceptable, with the exception of the \(^{147}\text{Sm} T_{20} \) measurements. This data set shows an unusual amount of scatter from angle to angle [2], and so large \( \chi^2 \) values are inevitable.

The minimization was performed for the higher-energy (7, 8, and 9 MeV) \(^{119}\text{Sn} \) data as well as for the sub-Coulomb data. We do not expect these higher-energy measurements to give an accurate result for \( \eta_t \), primarily because of the greatly increased sensitivity of the calculations to details of the optical model potentials. Although these higher-energy \((d,t)\) data are not useful in determining the value of \( \eta_t \), they do give us some insight into the error estimation procedure. We will return to this point.

### Table II. Values of \( \eta_t \) obtained from tensor analyzing power measurements for sub-Coulomb reactions.

<table>
<thead>
<tr>
<th>Target</th>
<th>( E_d ) (MeV)</th>
<th>( E_a ) (MeV)</th>
<th>( T_{92} )</th>
<th>( \eta_t )</th>
<th>( \chi^2 )</th>
<th>No. Pts. (%</th>
<th>C.L.</th>
<th>( \Delta \eta_t^S )</th>
<th>( \Delta \eta_t^C )</th>
<th>( \Delta \eta_t^N )</th>
<th>Weight (%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{91}\text{Zr} )</td>
<td>5.0</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0392</td>
<td>1.74</td>
<td>12</td>
<td>6</td>
<td>0.0025</td>
<td>0.0023</td>
<td>0.0008</td>
<td>6.2</td>
</tr>
<tr>
<td>(^{119}\text{Sn} )</td>
<td>6.0</td>
<td>0.00</td>
<td>( T_{21} )</td>
<td>-0.0459</td>
<td>0.89</td>
<td>12</td>
<td>54</td>
<td>0.0045</td>
<td>0.0021</td>
<td>0.0009</td>
<td>4.1</td>
</tr>
<tr>
<td>(^{147}\text{Sm} )</td>
<td>4.9</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0430</td>
<td>1.30</td>
<td>18</td>
<td>18</td>
<td>0.0012</td>
<td>0.0028</td>
<td>0.0003</td>
<td>10.7</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>10.0</td>
<td>0.00</td>
<td>( T_{21} )</td>
<td>-0.0419</td>
<td>0.92</td>
<td>12</td>
<td>52</td>
<td>0.0045</td>
<td>0.0029</td>
<td>0.0010</td>
<td>3.5</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>9.0</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0427</td>
<td>0.86</td>
<td>6</td>
<td>52</td>
<td>0.0027</td>
<td>0.0014</td>
<td>0.0003</td>
<td>11.2</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>0.90</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0384</td>
<td>1.06</td>
<td>12</td>
<td>39</td>
<td>0.0028</td>
<td>0.0042</td>
<td>0.0008</td>
<td>3.9</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>0.57</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0426</td>
<td>0.61</td>
<td>12</td>
<td>82</td>
<td>0.0029</td>
<td>0.0028</td>
<td>0.0009</td>
<td>6.2</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>0.90</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0457</td>
<td>0.94</td>
<td>12</td>
<td>50</td>
<td>0.0029</td>
<td>0.0023</td>
<td>0.0009</td>
<td>7.2</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>0.90</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0398</td>
<td>1.31</td>
<td>6</td>
<td>26</td>
<td>0.0011</td>
<td>0.0026</td>
<td>0.0008</td>
<td>11.7</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>0.90</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0404</td>
<td>0.38</td>
<td>6</td>
<td>86</td>
<td>0.0031</td>
<td>0.0032</td>
<td>0.0008</td>
<td>5.1</td>
</tr>
<tr>
<td>(^{208}\text{Pb} )</td>
<td>0.90</td>
<td>0.00</td>
<td>( T_{20} )</td>
<td>-0.0432</td>
<td>0.84</td>
<td>6</td>
<td>52</td>
<td>0.0016</td>
<td>0.0018</td>
<td>0.0009</td>
<td>15.9</td>
</tr>
</tbody>
</table>
FIG. 6. Results for \( \eta_i \) for the 14 sub-Coulomb cases in Table II. The values are plotted in the order in which they are listed in the table. The error bars shown are the statistical error bars only; calculational and normalization uncertainties are not shown. The solid line represents the average of the individual values, weighted by the total uncertainty (statistical, calculational, and normalization) for each value. The dashed line represents the total error of the average.

in Sec. V. Results for these higher-energy measurements are given in Table III. Note that as the deuteron energy increases above the Coulomb barrier (which is about 7 MeV for deuterons on \(^{119}\)Sn), the values of \( \eta_i \) determined from the data increase in magnitude.

D. Determination of the uncertainty

Systematic errors in the extraction of \( \eta_i \) can arise from uncertainties in the theoretical calculations of the \((d,t)\) tensor analyzing powers. The main contributions to this calculational error arise from uncertainties in the deuteron and triton elastic scattering wave functions (that is, in the optical model potentials) and to a lesser extent from uncertainties in the parameters used to specify the triton and bound-state neutron wave functions. We follow the method of Rodning and Knutson [29] to estimate the calculational uncertainty in \( \eta_i \). First, we assign an uncertainty \( \Delta p_i \) to each parameter in the calculation. We then carry out DWBA calculations in which each parameter is varied individually by its uncertainty \( \Delta p_i \) in order to determine the sensitivity of the extracted value of \( \eta_i \) to that parameter \((\delta \eta_i/\delta p_i)\). The net calculational uncertainty in \( \eta_i \), \( \Delta \eta_i^C \), is found by adding the contribution due to each of the parameters in quadrature:

\[
\Delta \eta_i^C = \left[ \sum_i \left( \frac{\delta \eta_i}{\delta p_i} \Delta p_i \right)^2 \right]^{1/2}.
\]

The difficult part of this procedure is assigning uncertainties to the parameters. As in [29], we fix the radius and diffuseness parameters for all the potentials, and assign to the depth an uncertainty large enough to cover the uncertainty in the geometry parameters.

For the central, absorptive, and spin-orbit potentials in the deuteron channel, we use uncertainties of \(\pm 30\%\), based on the analysis of Ref. [29]. We also use uncertainties of \(\pm 30\%\) in the potential depths in the triton channel, based on fits to \(^{208}\)Pb(\(t,t\))\(^{208}\)Pb cross section data at 9 MeV [32].

Another way to estimate the uncertainty due to the central and spin-orbit potentials is to repeat the DWBA calculations with different sets of optical model potentials, keeping the tensor potentials fixed at their original values. We did this for deuteron potentials from Daehnick et al. [15], Lohr [33], Lohr and Haeberli [34], Bojowald et al. [35], and Perrin et al. [36], and triton potentials from Becchetti and Greenlees [24], Hardekopf et al. [37], and Flynn et al. [38]. In this case, the uncertainty due to the central and spin-orbit potentials is taken to be the standard deviation of the set of resulting \( \eta_i \) values. For a given reaction, these uncertainties are up to three times larger than the uncertainties obtained from the method described in the previous paragraph. However, since the central and spin-orbit potentials contribute only a small amount to the total uncertainty for most of the reactions, the net effect on the final value of \( \Delta \eta_i^C \) is small.

The largest contributions to \( \Delta \eta_i^C \) arise from the deuteron-nucleus tensor potentials. Since a real or an imaginary potential, or a combination of both, could be used to fit the sub-Coulomb elastic scattering data, we choose uncertainties of \(\pm 100\%\) for both the real and the imaginary parts of the tensor potential. This means that the real and imaginary depths are allowed to vary independently from zero to the folding model value.

The uncertainty arising from the long-range tensor potentials is also obtained in the same way as in [29]. The error due to the quadrupole tensor term is negligible, since the quadrupole moment of the deuteron is known to high precision. To calculate the error due to the tensor polarizability term, we assign an uncertainty of \(\pm 10\%\) to the value of \( \tau \) calculated by Lopes.

To determine the uncertainty due to the wave function used for the bound-state neutron in the target nucleus, we assign uncertainties of \(\pm 15\%\) to the radius and diffuseness parameters. The spin-orbit potential depth in the neutron bound-state calculation was allowed to vary by \(\pm 30\%\).

For the triton wave function, uncertainties of 20\% in the radius and diffuseness parameters were used. This means, for example, that \( r \) is allowed to vary from 1.2 to 1.8 fm. The radial form factors \( v_0(r) \) and \( v_2(r) \) corresponding to these values of \( r \) are shown in Fig. 7. At the energies represented in our data set, the tensor analyzing powers depend mainly on the low-momentum behavior of the momentum-space form factors \( v_L(k) = \ldots \)
\[ J_0 \int_0^\infty j_L(kr) v_L(r) r^2 \, dr. \] Therefore, in order to emphasize the regions that are important for low momenta, the radial form factors that are multiplied by \( r^2 \). Also shown in Fig. 7 for comparison are the form factors for the Reid soft-core potential from the Faddeev calculation of Sasakawa and Sawada [39]. If the \((d, t)\) tensor analyzing powers do not depend only on the asymptotic \(D\)- to \(S\)-state ratio for the wave function, then the extracted \(\eta_t\) value may depend to some extent on the shape of the assumed form factors. Figure 7 demonstrates that the range of parameter values we employ, \( r = 1.5 \pm 0.3 \) fm and \( a = 0.5 \pm 0.1 \) fm, correspond to a wide range of form factor shapes, and that this range encompasses the Faddeev prediction.

The contributions to \(\Delta \eta_t^C\) from all these individual parameters are shown in Table IV for the case of \(^{208}\text{Pb}(d, t)^{207}\text{Pb}(E_x = 0.90 \text{ MeV})\) at \(E_d = 9.0 \text{ MeV}\). Most of \(\Delta \eta_t^C\) comes from the uncertainty in the nuclear tensor potentials. This is typical of the results for all reactions and energies in our data set.

To produce a value for the net uncertainty due to the theoretical calculation, the contributions from the individual components are added in quadrature. This result is shown in Table II as \(\Delta \eta_t^C\). An additional \( \pm 2\% \) has been added in quadrature to \(\Delta \eta_t^C\) to account for effects that were not investigated, such as virtual \(P\)-state effects, effects of channel coupling, and so on. Also shown in Table II are the uncertainties arising from statistical uncertainty \(\langle \Delta \eta_t^C \rangle\) and uncertainty in the overall normalization of the beam momenta \(\langle \Delta \eta_t^C \rangle\). An additional \(\pm 3.5\%\) was added in quadrature to \(\Delta \eta_t^C\) for the 4.9 MeV \(^{114}\text{Sn}\) result to account for the possible systematic errors in the choice of background subtraction method discussed in Sec. III. The beam moment normalization uncertainty is \(\pm 0.7\%\) [20] for the \(^{114}\text{Sn}(d, t)\) \(T_2\) data at 4.9 and 6.0 MeV, and \(\pm 2\%\) [18] for the old data and the rest of the new data.

\begin{table}
\centering
\caption{Contribution of individual parameters to the total calculational uncertainty for the reaction \(^{208}\text{Pb}(d, t)^{207}\text{Pb}(E_x = 0.90 \text{ MeV})\) at \(E_d = 9.0 \text{ MeV}\).}
\begin{tabular}{|c|c|c|}
\hline
Parameter \( (p_t) \) & \( \Delta p_t \) & \( \langle \Delta \eta_t^C \rangle \) \\
\hline
\( d \rightarrow ^{208}\text{Pb} \) & & \\
Real \( V \) & \( \pm 30\% \) & \( < 0.0001 \) \\
Surface imaginary \( V \) & \( \pm 30\% \) & 0.0002 \\
Spin-orbit \( V \) & \( \pm 30\% \) & \( < 0.0001 \) \\
Real tensor \( V \) & \( \pm 100\% \) & 0.0006 \\
Imaginary tensor \( V \) & \( \pm 100\% \) & 0.0013 \\
Quadrupole tensor \( V \) & \( \pm 10\% \) & \\
Polarization tensor \( V \) & \( \pm 10\% \) & 0.0002 \\
\hline
\( t \rightarrow ^{207}\text{Pb} \) & & \\
Real \( V \) & \( \pm 30\% \) & 0.0001 \\
Imaginary \( V \) & \( \pm 30\% \) & \( < 0.0001 \) \\
Spin-orbit \( V \) & \( \pm 30\% \) & \( < 0.0001 \) \\
\hline
\( ^{208}\text{Pb} \) bound state \( r, a \) & & \\
\( ^{208}\text{Pb} \) bound state \( r, a \) & \( \pm 20\% \) & 0.0005 \\
\( ^{208}\text{Pb} \) bound state \( r, a \) & \( \pm 15\% \) & 0.0002 \\
\hline
\end{tabular}
\end{table}

\section{V. Results}

In order to obtain a final value for \(\eta_t\), we have to decide how to weight the individual results. The procedure we use is to calculate the overall uncertainty

\[ \Delta \eta_t = \left[ (\Delta \eta_t^C)^2 + (\Delta \eta_t^{\text{geom}})^2 + (\Delta \eta_t^{\text{stat}})^2 \right]^{1/2} \]

for each result and then take the weighting factors to be inversely proportional to the square of these overall uncertainties. While other weighting procedures could have been adopted, we feel that the method described leads to the most reasonable distribution of weights for our data set. To take the large scatter of some of the measurements (particularly the \(^{147}\text{Sm} T_2\) measurements) into account, we have multiplied \(\Delta \eta_t^C\) by \(\sqrt{X_t^2}\) for those individual results for which \(\sqrt{X_t^2}\) is greater than 1. This gives these possibly less reliable measurements a smaller weight in determining the final result. The weights obtained in this way are given in Table II. The result of combining the individual \(\eta_t\) values using these weights is \(\eta_t = -0.0431\). We point out that if the measurements of Ref. [11] were included with our measurements in this
weighting process, the Ref. [11] measurements would receive about one-third of the total weight.

To obtain a value for the uncertainty in $\eta$, we need to take into account the fact that the uncertainties of the individual $\eta$ determinations are partially correlated. The statistical errors $\Delta \eta^S$ are completely uncorrelated, and the beam polarization normalization errors $\Delta \eta^N$ are uncorrelated except for measurements of the same analyzing power at the same energy. It is not known, however, how the errors in the calculation, $\Delta \eta^C$, are correlated. If an optical model parameter used in one calculation is incorrect, that parameter may or may not be incorrect for other energies or target nuclei. Also, different reactions have different sensitivities to the individual parameters. We have chosen to compute the uncertainty under the assumption that the calculational uncertainties are completely correlated (this is the worst-case assumption), while the statistical uncertainties are uncorrelated and the beam polarization normalization uncertainties are correlated to the extent mentioned above. When this is done, the result is $\Delta \eta = 0.0025$. The reduced $\chi^2$ for the set is 1.45 (corresponding to a confidence level of 13%), taking into account only the statistical errors. The final result of $\eta = -0.0431 \pm 0.0025$ is shown by the solid line and the dashed error bands in Fig. 6 along with the individual determinations of $\eta$.

To check our estimate of $\Delta \eta^C$, we can look at the results for the higher-energy $^{116}$Sn data. As pointed out earlier, we do not expect these results to be accurate, nor do we expect the calculation of $\Delta \eta^S$ as described above to be as meaningful as at sub-Coulomb energies. However, looking at these higher-energy data can give us a check on our error estimates. With the uncertainties (given in Table III) calculated in the same way as for the rest of the data, the higher-energy results lie within $1.5 \sigma$ of the weighted mean of the sub-Coulomb data. This indicates that the method of error estimation is probably reasonable.

VI. CONCLUSION

By comparing sub-Coulomb $(d, t)$ tensor analyzing powers to full finite-range DWBA calculations including tensor potentials, we have obtained a value for $\eta$ of $-0.0431 \pm 0.0025$. The quoted uncertainty includes statistical errors, errors in the theoretical calculations, and beam moment normalization errors. This value is in good agreement with the recent experimental determinations of Refs. [8, 11], and with the theoretical predictions of Refs. [3–5]. We believe that our result is the most accurate experimental determination of $\eta$ to date because of the large data set that was used, the low energy at which the measurements were made, and the care with which the uncertainty was estimated.

ACKNOWLEDGMENTS

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[37] R. A. Hardekopf, R. F. Haglund, Jr., G. G. Ohlsen, W. J. Thompson, and L. R. Veeser, Phys. Rev. C 21, 906 (1980). The potentials of Table II were used. For $^{118}$Sn, the potential for $^{116}$Sn was used; for $^{147}$Sm, the potential for $^{140}$Ce was used; for $^{207}$Pb, the potential for $^{208}$Pb was used.

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