Homework 10 Solutions

9-1 \[ \Delta E = 2 \mu_B B = hf \]
\[ 2(9.27 \times 10^{-24} \text{ J/T})(0.35 \text{ T}) = \left(6.63 \times 10^{-34} \text{ J s}\right) f \text{ so } f = 9.79 \times 10^9 \text{ Hz} \]

9-4

(a) 3d subshell \( \Rightarrow l = 2 \Rightarrow m_l = -2, -1, 0, 1, 2 \) and \( m_s = \pm \frac{1}{2} \) for each \( m_l \)

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<th>( n )</th>
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<th>( m_s )</th>
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(b) 3p subshell: for a \( p \) state, \( l = 1 \). Thus \( m_l \) can take on values \(-1\) to \(+1\), or \(-1, 0, 1\). For each \( m_l \), \( m_s \) can be \( \pm \frac{1}{2} \).

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9-9

With \( s = \frac{3}{2} \), the spin magnitude is \( |S| = |s(s+1)|^{1/2} \hbar = \left(\frac{15^{1/2}}{2}\right) \hbar \). The z-component of spin is \( S_z = m_s \hbar \) where \( m_s \) ranges from \(-s\) to \( s\) in integer steps or, in this case,

\[ m_s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2} \]. The spin vector \( S \) is inclined to the z-axis by an angle \( \theta \) such that

\[ \cos(\theta) = \frac{S_z}{|S|} = \frac{m_s \hbar}{\left(\frac{15^{1/2}}{2}\right) \hbar} = \frac{m_s}{\frac{15^{1/2}}{2}} = -\frac{3}{(15)^{1/2}}, -\frac{1}{(15)^{1/2}}, \frac{1}{(15)^{1/2}}, \frac{3}{(15)^{1/2}} \]

or \( \theta = 140.8^\circ, 105.0^\circ, 75.0^\circ, 39.2^\circ \). The \( \Omega^- \) does obey the Pauli Exclusion Principle, since the spin \( s \) of this particle is half-integral, as it is for all fermions.

9-11

For a \( d \) electron, \( l = 2; s = \frac{1}{2}; j = 2 + \frac{1}{2}, 2 - \frac{1}{2} \)

For \( j = \frac{5}{2}; m_j = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \)

For \( j = \frac{3}{2}; m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2} \)
9-13 (a) \(4f_{4/2} \rightarrow n=4, l=3, j=\frac{5}{2}\)

(b) \(|J| = [(j(j+1)]^{1/2} = \left[\left(\frac{5}{2}\right)^2\right]^{1/2} = \frac{35}{4}\)

(c) \(J_z = m_j h\) where \(m_j\) can be \(-j, -j+1, ..., j-1, j\) so \(m_j\) can be \(-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}\). \(J_z\) can be \(-\frac{5}{2} h, -\frac{3}{2} h, -\frac{1}{2} h, \frac{1}{2} h, \frac{3}{2} h, \) or \(\frac{5}{2} h\).

9-21 (a) \(1s^22s^22p^4\)

(b) For the two 1s electrons, \(n=1, l=0, m_l=0, m_s=\pm \frac{1}{2}\).
For the two 2s electrons, \(n=2, l=0, m_l=0, m_s=\pm \frac{1}{2}\).
For the four 2p electrons, \(n=2, l=1, m_l=1, 0, -1, m_s=\pm \frac{1}{2}\).

11-5 (a) The separation between two adjacent rotationally levels is given by \(\Delta E = \left(\frac{\hbar}{I}\right)^2\), where \(I\) is the quantum number of the higher level. Therefore

\[
\Delta E_{10} = \frac{\Delta E_{60}}{6} \\
\lambda_{10} = 6\lambda_{60} = 6(1.35 \text{ cm}) = 8.10 \text{ cm} \\
f_{10} = \frac{c}{\lambda_{10}} = \frac{3.00 \times 10^{10} \text{ cm/s}}{8.10 \text{ cm}} = 3.70 \text{ GHz}
\]

(b) \(\Delta E_{10} = \hbar f_{10} = \frac{\hbar^2}{I};
\)

\[
I = \frac{\hbar}{2\pi f_{10}} = \frac{1.055 \times 10^{-34} \text{ J.s}}{(2\pi)(3.70 \times 10^9 \text{ Hz})} = 4.53 \times 10^{-46} \text{ kg.m}^2
\]

(20) (a) The excitation energy is \(E = \ell(l+1) \frac{\hbar^2}{2I}\) where \(I = \mu R_0^2\) for CO the reduced mass is

\[
\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(12 u)(16 u)}{12 u + 16 u} = 6.957 u
\]

Here

\[
u_1 = 631.5 \text{ MeV}/c^2 = 9.315 \times 10^8 \text{ eV}/c^2
\]

So

\[
u_1, c^2 = 9.315 \times 10^8 \text{ eV}
\]

\[
\frac{\hbar^2}{2I} = \frac{(\hbar c)^2}{2 \mu c^2 R_0^2} = \left(\frac{1}{2\pi}\right)^2 \frac{(\hbar c)^2}{2 \mu c^2 R_0^2} = \left(\frac{1}{2\pi}\right)^2 \frac{(1240 \text{eV.nm})^2}{2(6.857)(9.315 \times 10^8 \text{ eV})(1,113 \text{ nm})^2}
\]
Thus for $l=1$

$$E_1 = (1)(2) \frac{\hbar^2}{2I} \Rightarrow E_1 = 4.775 \times 10^{-4} eV$$

$$E_2 = (2)(3) \frac{\hbar^2}{2I} \Rightarrow E_2 = 1.433 \times 10^{-3} eV$$

(b) We can find $N_e/N_0$ using the Boltzmann formula. Here

$$kT = (8.617 \times 10^{-5} eV/K)(290K) = 0.02499 eV$$

Thus

$$\frac{N_1}{N_0} = \frac{g_1}{g_0} e^{\frac{-E_1}{kT}} = (3) e^{\frac{-4.775 \times 10^{-4} eV}{2.499 \times 10^{-3} eV}} \Rightarrow \frac{N_1}{N_0} = 2.94$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_0} e^{\frac{-E_2}{kT}} = (5) e^{\frac{-1.433 \times 10^{-3} eV}{2.499 \times 10^{-3} eV}} \Rightarrow \frac{N_2}{N_0} = 4.72$$

(1.21) The absorbed photons have $\lambda = 3.69$ mm and therefore the energy is

$$E_0 = hf = \frac{hc}{\lambda} = (1240 eV \cdot nm)/(3.69 \times 10^6 nm) = 3.36 \times 10^{-4} eV.$$

Since

$$E_{rot} = \frac{\ell(\ell+1)\hbar^2}{2I}$$

the energy for the $l=0$ to $l=1$ transition is

$$E = E_1 - E_0 = \frac{(1)(2)\hbar^2}{2I} - 0 = \frac{\hbar^2}{I}.$$

Thus

$$E_0 = 3.36 \times 10^{-4} eV = \frac{\hbar^2}{I} \Rightarrow I = \frac{\hbar^2}{E_0}$$

but

$$I = \mu R_0^2 \quad \text{so} \quad R_0^2 = \frac{\hbar^2}{\mu E_0} = \frac{(hc)^2}{\mu c^2 E_0}.$$

$$\mu = \frac{m_e m_a}{m_e + m_a} = \frac{(7/10)}{7+19} u = 5.12 u \Rightarrow \mu c^2 = (5.12) (931.5 MeV) = 4.76 \times 10^9$$

Thus

$$R_0 = \frac{(hc)}{[\mu c^2 E_0]^{1/2}} = \frac{(1\pi)(1240 eV \cdot nm)/[(4.76 \times 10^9 MeV)(3.36 \times 10^{-4} eV)]}{[\mu c^2 E_0]^{1/2}}$$

$$\Rightarrow R_0 = 0.156 \text{ nm}.$$
The vibrational energy is \( E_{\text{vib}} = (\nu + \frac{1}{2}) h\omega_\nu \) and \( \nu = 0 \) to \( \nu = 1 \) for

\[ E_\nu = \frac{3}{2} h\omega_\nu - \frac{1}{2} h\omega_\nu = h\omega_0 \]

but \( E_\nu = hf \) so

\[ f = \frac{h\omega_0}{h} = \frac{\omega_0}{2\pi} \implies \omega_0 = 2\pi f \]

\[ \omega_0 = (2\pi)(5.63 \times 10^{13} \text{ s}) \implies \omega_0 = 3.54 \times 10^{14} \text{ s}^{-1} \]

(a) To find \( k \) we use \( \omega_0 = \sqrt{\frac{k}{m}} \implies k = \mu \omega_0^2 \)

\[ \mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(14u)(16u)}{14u + 16u} = 7.47 u = (7.47)(1.66 \times 10^{-27} \text{ kg}) = 1.24 \times 10^{-26} \text{ kg} \]

so

\[ k = \mu \omega_0^2 = (1.24 \times 10^{-26} \text{ kg})(3.54 \times 10^{14} \text{ s}^{-1})^2 \implies k = 1551 \text{ N/m} \]

(b) In the ground state the energy is \( E_0 = \frac{1}{2} h\omega_0 \). To find the amplitude of the vibrational motion, \( A \), we need to recall that \( E = K + V = \frac{1}{2} m_0 v^2 + \frac{1}{2} kx^2 \). When \( x \) is at its maximum value the kinetic energy is zero (the maximum \( x \) is the turning point so \( v \) must be zero) so

\[ \frac{1}{2} kA^2 = E = \frac{1}{2} h\omega_0 \]

Thus

\[ A = \left[ \frac{h\omega_0}{k} \right]^{\frac{1}{2}} \]

\[ A = \left[ \frac{(1.055 \times 10^{-34} \text{ J s})(3.54 \times 10^{14} \text{ s}^{-1})}{(1551 \text{ N/m})} \right]^{\frac{1}{2}} = 4.91 \times 10^{-12} \text{ m} \]

\[ A = 4.91 \times 10^{-3} \text{ nm} \]