1) A lens made of glass \((n = 1.57)\) is coated with a thin film \((n = 1.25)\). The goal is to minimize reflection of light at the center of the visible spectrum, \((\lambda_{\text{vac}} = 550\,\text{nm})\). Find the smallest film thickness for which this is accomplished.

We get reflected waves from the front surface and the back surface. Both waves are inverted on reflection, so they cancel if the pathlength difference is \(\frac{1}{2}\lambda\).

\[
2t = \frac{1}{2}\lambda = \frac{1}{2} \frac{\lambda_{\text{vac}}}{n}
\]

\[
t = \left(\frac{1}{4}\right) \frac{\lambda_{\text{vac}}}{1.25} = 110\,\text{nm}
\]

2) Light from a laser, initially unpolarized, illuminates a double slit. Both slits are covered with linear polarizers. With the transmission axes of the polarizers parallel to each other, you measure the light intensity, \(I\), at \(\theta = 0\). What will you get for the intensity at the same spot if you rotate one of the polarizers by \(90^\circ\)?

The wave passing through the polarizer has its electric field along the polarization axis. When the polarizations are parallel the two electric fields add like this.

When the polarizations are perpendicular we get this, so

\[
\mathbf{E}_\text{tot} = \sqrt{2} \mathbf{E}_0
\]

Since \(I \propto E^2\), the intensity is half in the second case.
3) When laser light is incident on a diffraction grating, the transmitted light will make spots on a screen as shown in the drawing. Suppose the laser produces light of two wavelengths, one blue and one green. Indicate on the drawing whether each of the spots will be blue or green (or mixed). Support your answers with work or an explanation.

4) An object is located some distance in front of a convex mirror. Determine the image location by constructing a ray diagram. Your diagram should be neat and accurate, and you should draw at least three primary rays.
5) An “opera glass” consists of a converging objective lens with \( f = 20 \text{ cm} \) and a diverging eyepiece lens with \( f = -8 \text{ cm} \). The lenses are separated by 11 cm as shown in the drawing.

(a) Determine the location and the properties of the final image formed by the opera glass when it is used to view a 1.5 m tall person standing 20 m away.

**Objective:** \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad p = 20 \text{ m} = 2000 \text{ cm} \)
\[ \frac{1}{q} = \frac{1}{20} - \frac{1}{2000} = \frac{q}{20} = 20.2 \text{ cm} \]
\[ m_1 = -\frac{q}{p} = -\frac{20.2}{2000} \]

**Eyepiece:** The image from the objective is behind the eyepiece, so
\[ p = -(20.2 - 11) = -9.2 \text{ cm} \]
\[ \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = -\frac{1}{8} - \frac{1}{-9.2} = \frac{1}{q} = 9.2 - \frac{1}{8} \Rightarrow q = -61.2 \text{ cm} \]
\[ m_2 = -\frac{\frac{q}{p}}{\frac{q}{p}} = -\frac{61.2}{-9.2} = 6.65 \]
\[ M_T = M_1 \cdot M_2 = 1067 \]

Explain where the image is located: **61.2 cm to the right of the eyepiece**

How tall is the image? \((1067) \times (150 \text{ cm}) = 10.1 \text{ cm}\)

Is the image **upright** or **inverted**?

Is the image **real** or **virtual**?

(b) Determine the magnifying power of the opera glass. (Remember that the magnifying power is a measure of the size of the image on your retina looking through the glass compared to looking at the object directly.)

Compare 10.1 cm at 61.2 cm with 1.5 m at 20 m

\[ \alpha = \frac{10.1}{61.2} \quad \alpha_0 = \frac{1.5}{20} \]
\[ m = \frac{\alpha}{\alpha_0} = \left(\frac{10.1}{61.2}\right)\left(\frac{20}{1.5}\right) = 2.2 \]
6) A glass rod \((n = 1.5)\) with a radius of 4 cm has ends that are hemispherical in shape. A light ray traveling parallel to the axis of the rod, 3 cm above the axis, enters the glass as shown. Find the distance, \(x\), from the point of entry to the point where the ray crosses the axis.

The dashed line is the normal to the surface, so Snell's Law gives

\[
(1.0) \sin \theta_1 = 1.5 \sin \theta_2
\]

We can get \(\theta_1\) from the geometry

\[
\sin \theta_1 = \frac{3 \text{ cm}}{4 \text{ cm}} = 0.75, \text{ so } \theta_1 = 48.6^\circ.
\]

\[
\sin \theta_2 = \left(\frac{1}{1.5}\right) \sin \theta_1 = \left(\frac{1}{1.5}\right) \left(\frac{3}{4}\right) = 0.5 \Rightarrow \theta_2 = 30^\circ
\]

\[
180^\circ - 48.6^\circ = 131.4^\circ
\]

\[
\alpha + 131.4^\circ + 30^\circ = 180^\circ \Rightarrow \alpha = 18.6^\circ
\]

\[
\tan \alpha = \frac{3 \text{ cm}}{x}
\]

\[
x = \frac{3}{\tan 18.6^\circ} = 8.92 \text{ cm}
\]

8.92 cm