**Homework 9 Solutions**

7-41)
(a) Cl \[ Z = 17 \] \((1s)^2(2s)^2(2p)^6(3s)^2(3p)^5\)
(b) Ca \[ Z = 20 \] \((1s)^2(2s)^2(2p)^6(3s)^2(3p)^6(4s)^2\)
(c) Ge \[ Z = 32 \] \((1s)^2(2s)^2(2p)^6(3s)^2(3p)^6(4s)^2(3d)^{10}(4p)^2\)

7-44)
The energy goes like \[ Z_{\text{eff}}^2 \] \( \Rightarrow \) \( E = -13.6 \text{ eV} \left( \frac{Z_{\text{eff}}^2}{n^2} \right) \)
So \[ Z_{\text{eff}} = \left( \frac{E}{-13.6} \right)^{\frac{1}{2}} n = \left( \frac{-5.14 \text{ eV}}{-13.6 \text{ eV}} \right)^{\frac{1}{2}} (3) = 1.84 \]

7-51)
The selection rules are \( \Delta l = \pm 1 \), \( \Delta j = 0, \pm 1 \)

4S_{1/2} \rightarrow 3S_{1/2} \hspace{1cm} \text{doesn't occur} \hspace{1cm} \text{violates } \Delta l \text{ rule } \hspace{1cm} (\Delta l = 0)
4S_{1/2} \rightarrow 3P_{3/2} \hspace{1cm} \text{OK}
4P_{3/2} \rightarrow 3S_{1/2} \hspace{1cm} \text{OK}
4D_{5/2} \rightarrow 3P_{1/2} \hspace{1cm} \text{doesn't occur} \hspace{1cm} \text{violates } \Delta j \text{ rule } \hspace{1cm} (\Delta j = 2)
4D_{3/2} \rightarrow 3P_{1/2} \hspace{1cm} \text{OK}
4D_{5/2} \rightarrow 3S_{1/2} \hspace{1cm} \text{doesn't occur} \hspace{1cm} \text{violates } \Delta l \text{ rule } \hspace{1cm} (\Delta l = 2)

7-54)
(a) The electron energy in the 3s state is \(-5.14 \text{ eV}\). From Fig. 7.22, the transitions from 3p to 3s have wavelengths of 588.99 and 588.59 nm \( \Rightarrow \) using the average 588.79 we have \( E = \frac{hc}{\lambda} = 1240 \text{ eV} \cdot \text{nm} / 588.79 \text{ nm} = 2.106 \text{ eV} \)

So \( E(3p) = -5.14 \text{ eV} + 2.106 \text{ eV} = -3.034 \text{ eV} \)

To get from 3p to 3d the wavelengths are 814.91 and 818.33 nm (average 816.16) \( \Rightarrow \) \( E = 1240 \text{ eV} \cdot \text{nm} / 816.16 \text{ nm} = 1.52 \text{ eV} \)
\( E(3d) = -3.034 \text{ eV} + 1.52 \text{ eV} = -1.516 \text{ eV} \)

\[ E(3s) = -5.14 \text{ eV} \hspace{1cm} E(3p) = -3.034 \text{ eV} \hspace{1cm} E(3d) = -1.516 \text{ eV} \]
(b) As in 7-44 above \[ Z_{\text{eff}} = \left( \frac{E}{-13.6 \text{eV}} \right)^{\frac{1}{2}} \cdot n \]

\[ 3\text{s} \Rightarrow Z_{\text{eff}} = 1.84 \]
\[ 3\text{p} \Rightarrow Z_{\text{eff}} = 1.42 \]
\[ 3\text{d} \Rightarrow Z_{\text{eff}} = 1.002 \]

(c) \(-13.6 \text{eV}/n^2\) is pretty good for the 2s state.

7-63) (a) \[ E = \frac{\hbar c}{\lambda} \]
\[ E_1 = \frac{1240 \text{eV} \cdot \text{nm}}{766.41 \text{nm}} = 1.6174 \text{eV} \]
\[ E_2 = \frac{1240 \text{eV} \cdot \text{nm}}{769.90 \text{nm}} = 1.6106 \text{eV} \]

(b) \[ \Delta E = 1.6174 \text{eV} - 1.6106 \text{eV} = 0.0073 \text{eV} \]

(c) The electron's magnetic moment is \( \mu = -g_s \cdot \frac{e}{2m} \cdot S \), so \( \mu_z = -g_s \cdot \frac{e}{2m} \cdot (\pm \frac{1}{2} \hbar) \) for \( S_z = m_s \) with \( m_s = \pm \frac{1}{2} \). The magnetic energy is \( V_{\text{mag}} = -\mu \cdot B \) so for \( B \) along the z axis

\[ V_{\text{mag}} = g_s \cdot \frac{e}{2m} (\pm \frac{1}{2} \hbar) B = \pm \frac{9}{2} \left( \frac{e \hbar}{2m} \right) B \]

The energy difference between the two states is then
\[ \Delta V = 2 \left( \frac{9}{2} \right) \left( \frac{e \hbar}{2m} \right) B = g_s \mu_B B \]

Using \( g_s = 2 \), \( \mu_B = 9.274 \times 10^{-24} \text{J/T} = 5.79 \times 10^{-5} \text{eV/T} \)
we get

\[ B \approx \frac{0.0073 \text{eV}}{(2)(5.79 \times 10^{-5} \text{eV/T})} = 63 \text{ Tesla} \]

E-17) (a) At \( T=0 \), we put the particles into the lowest possible energy states. Fermions satisfy the Pauli principle, so we can only have one in each quantum state \( \Rightarrow 2 \) at each energy. Bosons can occupy the same state so at \( T=0 \) all will be in the ground state.
(b) Here we use the formulas to calculate $n_i$

$$n_i = \frac{g_i}{e^{\frac{E_i}{kT}} + 1} \quad \text{where} \quad E_i = (i + \frac{1}{2})hw_0 + kT = \frac{3}{2}hw_0$$

Here are the $n_i$ values:

<table>
<thead>
<tr>
<th>$i$</th>
<th>Bosons</th>
<th>Fermions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.626</td>
<td>1.902</td>
</tr>
<tr>
<td>1</td>
<td>1.371</td>
<td>1.817</td>
</tr>
<tr>
<td>2</td>
<td>0.528</td>
<td>1.673</td>
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<tr>
<td>3</td>
<td>0.240</td>
<td>1.448</td>
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<tr>
<td>4</td>
<td>0.117</td>
<td>1.148</td>
</tr>
<tr>
<td>5</td>
<td>0.058</td>
<td>0.817</td>
</tr>
<tr>
<td>6</td>
<td>0.029</td>
<td>0.524</td>
</tr>
<tr>
<td>7</td>
<td>0.015</td>
<td>0.308</td>
</tr>
<tr>
<td>8</td>
<td>0.008</td>
<td>0.171</td>
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<tr>
<td>9</td>
<td>0.004</td>
<td>0.092</td>
</tr>
<tr>
<td>10</td>
<td>0.002</td>
<td>0.048</td>
</tr>
<tr>
<td>11</td>
<td>0.001</td>
<td>0.025</td>
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<tr>
<td>12</td>
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<td></td>
</tr>
<tr>
<td>13</td>
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<tr>
<td>14</td>
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</tr>
<tr>
<td>15</td>
<td>0.002</td>
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</tr>
<tr>
<td></td>
<td>9.998</td>
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</tr>
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</table>
8-18) Although it doesn't say, I think we are supposed to assume room temperature, \( \Rightarrow \) \( kT \approx 0.025 \text{eV} \). We are also assuming that the density is low enough so that we can use Maxwell-Boltzmann statistics. Then the formulas 8-68 apply:

\[
\frac{N}{V} = 2 \left( \frac{2\pi m kT}{\hbar^2} \right)^{3/2} e^{-\frac{x}{kT}} = 2 \left( \frac{2\pi m c^2 kT}{(hc)^2} \right)^{3/2} e^{-\frac{x}{kT}}
\]

(a) \[
\frac{N}{V} = 2 \left[ \frac{2\pi (511 \times 10^5 \text{eV})(0.025 \text{eV})}{(1240 \text{eV} \cdot \text{nm})^2} \right]^{3/2} = 2.4 \times 10^{-2} / \text{nm}^3
\]

(b) For \( x/kT = 10^{-6} \), we get \( \frac{N}{V} = 2.4 \times 10^{19} / \text{m}^3 \)

9-2) From Table 9-2, the dissociation energy of NaCl is 4.27 eV. Suppose we do \( 2 \text{Na} + \text{Cl}_2 \Rightarrow 2 \text{NaCl} \).

It requires \( 2.48 \text{ eV} \) to dissociate the \( \text{Cl}_2 \), and then we gain \( 4.27 \text{ eV} \), making each \( \text{NaCl} \), so the net energy gain is \( 2 \times 4.27 \text{ eV} - 2.48 \text{ eV} = 6.06 \text{ eV} \).

The reaction is exothermic.

The energy released per \( \text{NaCl} \) molecule produced is \( 3.03 \text{ eV} \).

9-5) (a) \[
E = \frac{e^2}{4\pi \varepsilon_0 r} = \frac{1.44 \text{ eV} \cdot \text{nm}}{0.267 \text{ nm}} = +5.39 \text{ eV}
\]

(b) To make \( \text{KCl} \) out of \( \text{K} \) and \( \text{Cl} \), we supply \( 4.34 \text{ eV} \) to ionize \( \text{K} \), get back \( 3.62 \text{ eV} \) attaching the electron to \( \text{Cl} \) and gain \( 5.39 \text{ eV} \) from attraction, so the net gain would be \( -4.34 \text{ eV} + 3.62 \text{ eV} + 5.39 \text{ eV} = 4.67 \text{ eV} \).

Neglecting repulsion, we predict a dissociation energy of \( 4.67 \text{ eV} \).

(c) The measured energy is \( 4.43 \text{ eV} \), so the energy from repulsion must be \( 0.24 \text{ eV} \).

9-12) For purely ionic bond \( p = (1.602 \times 10^{-19} \text{C}) \cdot (0.0917 \text{ nm}) = 1.47 \times 10^{-29} \text{ C} \cdot \text{m} \). % ionic = \( \frac{6.4 \times 10^{-30}}{1.47 \times 10^{-29}} \times 100 = 43.6\% \)