20) (a) There are 10 single-electron states so \[ N = \frac{10 \times 9}{2} = 45 \]

We can have \( S = 0 \) (antisymmetric) or \( S = 1 \) (symmetric). The allowed \( l \) values are \( l = 4, 3, 2, 1, 0 \). Here \( l = 4, 2, 0 \) will be symmetric and \( l = 3, 1 \) will be antisymmetric. To get an antisymmetric wave function the choices are \( 'G', '3F', '1D', '3P' \) and \( 'S' \). Now \( J = \pm \frac{3}{2} \) gives:

\[ \begin{align*}
  &\{ 'G_4, '3F_2, '3F_3, '3F_4, '1D_2, '3P_0, '3P_1, '3P_2, 'S_0 \} \\
  \text{Total} &\; 9 + 5 + 7 + 9 + 5 + 1 + 3 + 5 + 1 = 45
\end{align*} \]

(b) There are 10 choices for the 3d electron and 10 for the 4d electron \( \Rightarrow N = 100 \)

Here we get all possible \( l, s \) combinations \( 'G', '3G', '1F', '3F' \) etc:

\[ \begin{align*}
  &\{ 'G_4, '3G_2, '3G_3, '3G_5, '1F_3, '3F_2, '3F_3, '3F_4, '1D_2, '3P_0, '3P_1, '3P_2, '1S_0, '3S_1 \} \\
  \text{Total} &\; 9 + 7 + 9 + 11 + 7 + 9 + 5 + 7 + 3 + 1 + 3 + 5 + 1 + 3 = 100
\end{align*} \]

(c) 6 choices for \( 4p \) 14 choices for \( 4F \) \( \Rightarrow 84 \)

\( S = 0 \) or \( 1 \) \( l = 2, 3, 4 \) all combinations allowed:

\[ \begin{align*}
  &\{ 'G_4, '3G_3, 'G_4, '3G_5, '1F_3, '3F_2, '3F_3, '3F_4, '1D_2, '3P_0, '3P_1, '3P_2, '1S_0, '3S_1 \} \\
  \text{9 + 7 + 9 + 11 + 7 + 5 + 7 + 9 + 5 + 3 + 5 + 7} &\; = 84
\end{align*} \]

(d) For \( (2p)^2 \) there are 15 possibilities and for \( (3p)^1 \) there are 6 so we should get \( N = 15 \times 6 = 90 \)

For the \( (2p)^2 \) electrons we 3 possible \( l, s \) combinations,

i) \( l = 0, s = 0 \) ii) \( l = 1, s = 1 \) iii) \( l = 2, s = 0 \) where the
quantum numbers are for $L_1 + L_2$ and $S_1 + S_2$. So now we add $l_3 = 1$ and $s_3 = \frac{k}{2}$.

(i) $l = 0, l_3 = 1 \Rightarrow l_{\text{tot}} = 1$

$s = 0 \quad s_3 = \frac{1}{2} \Rightarrow s_{\text{tot}} = \frac{k}{2} \Rightarrow$

\[2p_{\frac{3}{2}}, \frac{2}{2}p_{\frac{1}{2}}, \frac{4}{2}p_{\frac{1}{2}}, \frac{4}{2}p_{\frac{3}{2}}; \frac{4}{2}p_{\frac{3}{2}}, 2d_{\frac{1}{2}}, 2d_{\frac{3}{2}}, 2d_{\frac{5}{2}}

(ii) $l = 1, l_3 = 1 \Rightarrow l_{\text{tot}} = 0, 1, 2$

$s = 1 \quad s_3 = \frac{1}{2} \Rightarrow s_{\text{tot}} = \frac{k}{2}, \frac{3}{2} \Rightarrow$

\[2s_{\frac{1}{2}}, 4s_{\frac{1}{2}}, 2p_{\frac{1}{2}}, 2p_{\frac{3}{2}}, 4p_{\frac{1}{2}}, 4p_{\frac{3}{2}}, 4p_{\frac{5}{2}}, 4p_{\frac{5}{2}}; 2d_{\frac{1}{2}}, 2d_{\frac{3}{2}}, 2d_{\frac{5}{2}}, 2d_{\frac{7}{2}}

(iii) $l = 2, l_3 = 1 \Rightarrow l_{\text{tot}} = 1, 2, 3$

$s = 0 \quad s_3 = \frac{1}{2} \Rightarrow s_{\text{tot}} = \frac{k}{2} \Rightarrow$

\[2p_{\frac{1}{2}}, 2p_{\frac{3}{2}}, 2d_{\frac{1}{2}}, 2d_{\frac{3}{2}}, 2d_{\frac{5}{2}}, 2f_{\frac{5}{2}}, 2f_{\frac{7}{2}}

Total states = 2 + 4 + 2 + 4 + 2 + 4 + 2 + 4 + 2 + 4 + 6 + 8

+ 2 + 4 + 4 + 6 + 6 + 8 = 90

21) The states are $^1G_4, ^3F_2, ^3F_3, ^3F_4, ^1D_2, ^3P_0, ^3P_1, ^3P_2, ^1S_0$

Rule #1: Higher $S \Rightarrow$ lower energy $\Rightarrow$ triplets lower than singlets

Rule #2: Higher $l \Rightarrow$ lower energy $\Rightarrow$ $F$ below $P,$ $G$ below $D,$ below $S.$

Rule #3: Lower $j \Rightarrow$ lower energy.

Lowest to highest:

$3F_2, 3F_3, 3F_4; 3P_0, 3P_1, 3P_2; ^1G_4, ^1D_2, ^1S_0.$
22) For a $^4D$ state $l=2$, $s=\frac{3}{2}$ $\Rightarrow j=\frac{7}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$

According to the interval rule the splitting between 2 states is proportional to the $j$ of the higher one.

$\Delta E(\frac{7}{2} - \frac{5}{2}) = \frac{7}{2} \text{ C}$

$\Delta E(\frac{5}{2} - \frac{3}{2}) = \frac{5}{2} \text{ C}$

$\Delta E(\frac{3}{2} - \frac{1}{2}) = \frac{3}{2} \text{ C}$

From the picture with my definitions for the $\Delta E$'s

$$\frac{\Delta E_1}{\Delta E_0} = \frac{5}{3} \quad \frac{\Delta E_2}{\Delta E_0} = \frac{7}{3}$$

23) $^4F_{3/2} \Rightarrow l=3$, $s=\frac{3}{2}$, $j=\frac{3}{2}$

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} = 1 + \frac{\frac{3}{2}(\frac{3}{2}) + \frac{3}{2}(\frac{5}{2}) - 3(4)}{2 \cdot \frac{3}{2} \cdot \frac{5}{2}}$$

$$g = 1 + \frac{15 + 15 - 48}{30} = 1 - \frac{18}{30} = 1 - \frac{3}{5} = \frac{2}{5}$$

$^4D_{5/2}$ $l=2$, $s=\frac{3}{2}$, $j=\frac{5}{2}$ $\Rightarrow$ $g = 1 + \frac{\frac{5}{2}(\frac{5}{2}) + (\frac{3}{2})(\frac{3}{2}) - 2(3)}{2(\frac{5}{2})(\frac{3}{2})}$

$$g = 1 + \frac{25 + 15 - 24}{70} = 1 + \frac{26}{70} = \frac{48}{35}$$

The energy shifts are given $E^{(n)} = g\mu B m_j$ so we get

$$^4D_{5/2}, g\mu B = \frac{48}{35}(5.788 \times 10^{-5} \text{ eV/T})(0.8 \text{ T}) = \frac{6.35 \times 10^{-5} \text{ eV}}{}

$$^4F_{3/2} \quad = \quad (\frac{7}{5}) \quad = \quad 1.85 \times 10^{-5} \text{ eV}$$
For the emission we have

\[ E_y = E_0 + (m_i g_i - m_f g_f) \mu_0 B \]

The selection rule is \( \Delta m = 0, \pm 1 \) so

for each \( ^4F_{3/2} \) final state there are 3 possible \( ^4G_{s/2} \) initial states \( \Rightarrow \) 12 transitions in all.

Given \( E_y \) we calculate the wavelength by \( E = \frac{hc}{\lambda} \)

\[ \lambda = \frac{hc}{E} \quad \text{All the energies are close to } E_0 \quad (E = E_0 + \delta E) \]

and so the shift in wavelength is

\[ \delta \lambda \approx \frac{\delta \lambda}{\delta E} \delta E = -\frac{hc}{E^2} \delta E = -\frac{hc}{E_0 E_0} \delta E = -\lambda_0 \frac{\delta E}{E_0} \]

\[ \lambda = \lambda_0 + \delta \lambda \approx \lambda_0 \left(1 - \frac{\delta E}{E_0}\right) \quad \text{where } \lambda_0 = 375 \text{nm} \]

and \( E_0 = \frac{hc}{\lambda_0} = 1240 \text{eV}\cdot\text{nm} / 375 \text{nm} = 3.307 \text{eV} \)

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