1 Conceptual Exercises

10.) “A train moves eastward at 20 m/s. Velma, standing inside the train, throws a ball at 20 m/s toward the rear of the train. Find the ball’s velocity relative to Mort, who is standing beside the tracks.”

Galilean relativity says that we simply add or subtract the velocities, depending on whether the ball is thrown in the direction or opposite the direction the train is moving. Since Velma throws the ball opposite the direction the train is moving, we subtract the velocities. Thus, Mort observes a speed of

\[ 20 \text{ m/s} - 20 \text{ m/s} = 0 \text{ m/s}. \]

16.) “Although everything is normal inside a smoothly moving jet airplane, things are not normal when the ride is bumpy; you spill your coffee, you experience moments of partial weightlessness, and so on. Does this conflict with Einstein’s principle of relativity? Explain.”

Einstein’s principle of relativity is “every nonaccelerated observer observes the same laws of nature. In other words, no experiment performed within a sealed room moving at an unchanging velocity can tell you whether you are standing still or moving.”

When you experience jolts and bumps on a plane, clearly you can tell that you are moving and not standing still. This does not conflict with Einstein’s relativity because during these moments the plane is being accelerated. It is these small accelerations that inform you that you are in motion.

20.) “If you drop a coin inside a car that is slowing down, where will the coin land?”
When you drop the coin it is moving with the same horizontal speed as the car; as the car slows down, the coin, not being attached to anything, will continue moving with the original horizontal speed (because of inertia, an object in motion will stay in motion), while the car is now moving slower. Thus, the coin will appear to fall forward, landing in front of where it was dropped.

28.) “Velma’s rocket ship is moving away from Mort at 0.75c. She fires a laser beam in the backward direction, toward Mort. According to Galilean relativity, how fast does the laser beam move relative to Mort, assuming that Velma observes the beam to move away from her at speed c? How fast according to Einstein’s relativity?”

A laser is light, and light does not have special treatment in Galilean relativity, so we should subtract the velocities (see Concept #10). Thus, 

Mort would observe the laser beam to move at $c - 0.75c = 0.25c$.

According the Einstein, light always has the same speed for all observers, namely the speed of light. Thus, Mort would observe the laser beam to move at $c = 3 \times 10^8$ m/s.

34.) “The center of our galaxy is about 30,000 light-years away. Could a person possibly travel there in less than 30,000 years as measured on Earth? Could a person possibly travel there in less than 30,000 years of his or her own time? Explain.”

Since light is the ultimate speed limit, the fastest someone could get to the other galaxy according to someone on Earth is a little more than 30,000 years (traveling close to the speed of light). The person on Earth would notice, however, that time on the spaceship was running slower, so the space traveler could possibly make it in less than 30,000 years of their own time.

Consider, also, the situation from the frame of the space traveler. To them, their time does not run slow, but instead runs at the same rate as always. So how do they make it to the far away galaxy in less than 30,000 years? The key concept is length contraction - to them, the entire distance from them to the galaxy is moving past them near the speed of light, so the 30,000 light-year distance is now contracted; the less distance to travel, the shorter amount of time it will take according to the space traveler.

## 2 Problems

4.) “Velma passes Mort at 150,000 km/s. What fraction of lightspeed is this? What is the duration of one of Mort’s seconds (a time interval that Mort observes to be on second in duration) as observed by Velma?”
Divide the speed by \( c = 3 \times 10^8 \text{ m/s} \) to find the fraction of lightspeed (remember to convert to m/s!!!):

\[
\frac{150,000 \text{ km/s}}{3 \times 10^8 \text{ m/s}} = \frac{1.5 \times 10^5 \text{ m/s}}{3 \times 10^8 \text{ m/s}} = .5.
\]

Thus, their relative speed is .5c.

To find the duration of one of Mort’s seconds according to Velma, we should first note that we are seeing the situation from Velma’s frame, where Velma is stationary and Mort is moving past her at \( s=.5c \). Thus, Mort’s clocks should be running slow (Mort is the proper one), so the answer we get should be larger than 1 sec. To figure out the actual time, we must use:

\[
T = T_0/\sqrt{1 - s^2/c^2}.
\]

Since we are observing Mort’s watch in another frame, and Mort is observing his watch in his frame, we use \( T_0 = 1\text{ sec} \) (notice that this will give an answer larger than 1 sec). Thus, we have,

\[
T = 1\text{ sec} \cdot \frac{1}{\sqrt{1 - s^2/c^2}} = \frac{1\text{ sec}}{\sqrt{1 - (.5c)^2/c^2}} = \frac{1\text{ sec}}{\sqrt{1 - (.5)^2}} = 1.15 \text{ sec}. \tag{1}
\]

6.) “Velma passes Mort at a high speed. Her clock, as observed by him, runs at 25/of its normal speed - for example, his clock advances by only 15 minutes during a time of one hour as recorded on her own clock. What must be the value of the quantity \( \sqrt{1 - s^2/c^2} \)? Find Velma’s speed relative to Mort.”

Once again, we use the equation \( T = T_0/\sqrt{1 - s^2/c^2} \); here, since Mort is observing Velma’s clock from another frame, we use \( T_0 = 15\text{ min} \) and \( T = 1\text{ hr} \). Thus, we have \( \sqrt{1 - s^2/c^2} = .25 \). Solving for \( s \), we have \( s = \sqrt{15/16} c = .97c \).