**Hour Exam 3 Review**

- **Hour Exam 3**: Wednesday, Apr. 18 In-class
- Twenty multiple-choice questions
- Will cover: Basic Quantum Mechanics
- Uses of Quantum Mechanics
- The nucleus, radioactive decay
- This is Lectures 22-31, Chap. 13-15.5 in text
- You should bring
  - Your student ID
  - 1 page notes, written double sided
  - Calculator
  - Pencil for marking answer sheet

**From last time...**

- Radioactive decay
  - Unstable nuclei change to a more stable configuration by emitting a particle
  - Heavy nuclei can emit alpha particle (helium nucleus)
  - Lighter atoms can emit beta particle (electron)

**Beta decay**

- Nucleus emits an electron (negative charge)
- Must be balanced by a positive charge appearing in the nucleus.

**How can this be?**

- Nucleons have an internal structure.
- Made up of different types of quarks.
- In this sense, neutrons and protons not so different.

**Neutron decay**

- Can be clearer to visualize this diagrammatically.
- One of the down quarks changed to an up quark.
- We identify this new combination of quarks as a proton.

**Example of beta decay**

- $^{14}\text{C}$ (radioactive form of carbon) decays by beta decay (electron emission).
- Carbon has 6 electrons, so six protons.
- $^{14}\text{C}$ has $(14-6)=8$ neutrons.

Now have a new element (one more proton)
Element with 7 protons in nucleus is Nitrogen

$$^{14}\text{C} \rightarrow ^{14}\text{N} + e^-$$

Beta decay
- number of nucleons stays fixed, but
- one of the neutrons changes into a proton
- one additional proton -> different element!
Radioactive carbon
• $^{14}$C naturally present in the atmosphere as a result of transmutation of $^{14}$N.

Cosmic ray proton shatters nucleus of atmospheric gas atom. This produces neutrons.
Neutron knocks proton out of $^{14}$N nucleus.
$^{14}$N becomes $^{14}$C after losing neutron.

Natural atmospheric abundance
• $^{14}$C is produced at a particular rate from transmutation of $^{14}$N in upper atmosphere.
• $^{14}$C is radioactive with half-life of 5,730 yrs.
  - Decays at rate set by half-life
  - Balance of these results in an equilibrium ratio $\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12}$ in the atmosphere
• One radioactive $^{14}$C atom for every $\sim 10^{12}$ non-radioactive $^{12}$C atom.

$^{14}$C to $^{12}$C ratio
• $^{14}$C has a half-life of ~6,000 years, continually decaying back into $^{14}$N.
• Steady-state achieved in atmosphere, with $^{14}$C:$^{12}$C ratio ~ 1:1 trillion (1 part in $10^{12}$).

As long as biological material alive, atmospheric carbon mix ingested (as CO$_2$), ratio stays fixed.
After death, no exchange with atmosphere. Ratio starts to change as $^{14}$C decays.

Carbon-dating question
The $^{14}$C:$^{12}$C ratio in a fossil bone is found to be 1/8 that of the ratio in the bone of a living animal. The half-life of $^{14}$C is 5,730 years. What is the approximate age of the fossil?

A. 7,640 years
B. 17,200 years
C. 22,900 years
D. 45,800 years

Since the ratio has been reduced by a factor of 8, three half-lives have passed.
$3 \times 5,730 \text{ years} = 17,190 \text{ years}$

Other carbon decays
• Lightest isotopes of carbon are observed to emit a particle like an electron, but has a positive charge!
  - This is the antiparticle of the electron.
  - Called the positron.

Antimatter
• Every particle now known to have an antiparticle.
• Even antimatter has been generated.

Matter and antimatter annihilate when in close proximity.
Photons are created so that energy is conserved.
What is going on?

- $^{14}$C has more neutrons than the most stable form $^{12}$C.
  - So it decays by electron emission, changing neutron into a proton.
- Other isotopes of carbon have fewer neutrons.
  - Decays by emitting positron, changing proton into neutron.

Decay question

$^{20}$Na decays into $^{20}$Ne, a particle is emitted? What particle is it?

Na atomic number = 11
Ne atomic number = 10

A. Alpha
B. Electron beta
C. Positron beta
D. Gamma

Gamma decay

- So far
  - Alpha decay: alpha particle emitted from nucleus
  - Beta decay: electron or positron emitted
- Both can leave the nucleus in excited state
  - Just like a hydrogen atom can be in an excited state
  - Hydrogen emits photon as it drops to lower state.

Radioactive decay summary

- Alpha decay
  - Heavy nucleus emits alpha particle (2 protons + 2 neutrons)
- Beta decay
  - Beta $-^-$: Nucleus emits electron (neutron becomes proton)
  - Beta $+^+$: Nucleus emits positron (proton becomes neutron)
- Gamma decay
  - Nucleus emits high-energy photon and drops from excited state to lower energy state.

Review: Quantum Mechanics

- Quantization of light
  - Light comes in discrete clumps (photons)
  - Light shows both particle and wave-like properties
  - Photon energy $E = hf = hc/\lambda$.
- Matter waves
  - Matter shows both particle and wave-like properties
    - de Broglie wavelength $= \frac{h}{p} = \frac{Planck's constant}{momentum}$
  - Evidence for wave properties is interference and diffraction

Photoelectric effect summary

- Light is made up of photons, individual “particles”, each with energy $E = hf = \frac{hc}{\lambda}$.
- One photon collides with one electron
  - Knocks it out of metal.
  - If photon doesn’t have enough energy, cannot knock electron out.
- Intensity ( = # photons / sec) doesn’t change this.
**Photoelectric effect question**
A scientist is trying to eject electrons from a metal by shining a light on it, but none are coming out. To eject electrons, she should change the light by...

- A. decreasing the frequency
- **B. increasing the frequency**
- C. increasing the intensity
- D. increasing the wavelength

**Minimum frequency (maximum wavelength) required to eject electron**

**Compton scattering**
- Collision of photon and electron in vacuum
- Photon loses energy, transfers it to electron
- Photon loses momentum transfers it to electron
- Total energy and momentum conserved

**Compton scattering question**
A green photon collides with a stationary electron. After the collision the photon color is

- A. unchanged
- B. shifted toward red
- **C. shifted toward blue**

**Topic: Wave properties of matter**
- All objects show both wave-like properties and particle-like properties.
- Electromagnetic radiation (e.g. light) shows interference effects (wave-like properties), but also comes in discrete photons of energy \( hf \) (particle-like properties)
- Matter clearly shows particle-like properties, but also shows interference and diffraction effects (wave-like properties).

**Matter waves**
- If light waves have particle-like properties, matter should have wave properties.
- de Broglie postulated that the wavelength of matter is related to momentum as

\[
\lambda = \frac{h}{p}
\]

- This is called the de Broglie wavelength.

**Matter wave question**
A neutron has almost 2000 times the rest mass of an electron. Suppose they both have 1 ev of energy. How do their wavelengths compare?

- A. both same
- **B. neutron wavelength < electron wavelength**
- C. neutron wavelength > electron wavelength

Wavelength depends on momentum, as \( h/p \). Same momentum \( \rightarrow \) same wavelength. Momentum \( = \sqrt{2mE} \), depends on energy AND mass.
Using Quantum Mechanics
- Quantum states in a hydrogen atom
  - Models of the hydrogen atom
  - Absorption and emission of light (line spectra)
- The wavefunction of a quantum state
  - The ground state and excited states
  - Probabilistic interpretation of the wavefunction.
- Heisenberg uncertainty principle
  - Position and momentum cannot be known simultaneously
  - Consequence of wave properties

Quantum systems
- All quantum systems have discrete allowed “configurations”.
- Each has at least some distinguishing property
  - Energy
  - Momentum
  - Wavefunction (spatial probability)
  - Spin
- If states have different energy, can move from one state to another by absorbing/emitting energy

Topic: The hydrogen atom
- Hydrogen atom:
  - One electron orbiting around one proton (nucleus)
  - Electron can be in different “quantum states”
  - Quantum states determined by wave condition
  - Quantum states labeled by integer \( n \)
  - In each different quantum state, electron has
    - Different orbital radius
    - Different energy
    - Different wavelength
  - \( n=1 \) is lowest energy state, energy depends on state as \( -\frac{13.6}{n^2} \text{ eV} \)

Electron standing-waves on an atom
- Electron wave extends around circumference of orbit.
- Only integer number of wavelengths around orbit allowed.

Energy levels
- Instead of drawing orbits, we can just indicate the energy an electron would have if it were in that orbit.

Emitting and absorbing light
- Photon is emitted when electron drops from one quantum state to another
- Absorbing a photon of correct energy makes electron jump to higher quantum state.
Line spectra

- This says that gases such as Hydrogen emit light only at certain frequencies, wavelengths.
- The photon energies correspond to separations between the energy levels.

Wavelength (nm)

Spectral Question

Compare the wavelength of a photon produced from a transition from n=3 to n=1 with that of a photon produced from a transition n=2 to n=1.

A. $\lambda_{31} < \lambda_{21}$
B. $\lambda_{31} = \lambda_{21}$
C. $\lambda_{31} > \lambda_{21}$

$E_{31} > E_{21}$ so $\lambda_{31} < \lambda_{21}$

Topic: The wavefunction

- Particle can exist in different quantum states, having
  - Different energy
  - Different momentum
  - Different wavelength
- The quantum wavefunction describes wave nature of particle.
- Square of the wavefunction gives probability of finding particle.
- Zero’s in probability arise from interference of the particle wave with itself.

Particle in a box: Wavefunctions

Wavefunction

Probability $= (\text{Wavefunction})^2$

- Ground state wavefunction and probability.
- Height of probability curve represents likelihood of finding particle at that point.

Probability of finding electron

- Classically, equally likely to find particle anywhere
- QM - true on average for high N

Quantum Corral

- 48 iron atoms assembled into a circular ring.
- The ripples inside the ring reflect the electron quantum states of a circular ring (interference effects).
**Wavefunction question**

For this wavefunction, at what point is the probability of finding the particle the smallest?

A. A  
B. B  
C. C  
D. D

**Wavefunction: Particle in a Box**

- In Quantum Mechanics, ball represented by wave
  - Wave reflects back and forth from the walls.
  - Reflections cancel unless wavelength meets the standing wave condition: 
    integer number of half-wavelengths fit in the tube.

\[
\lambda = L \\
\text{Two half-wavelengths}
\]

\[
\lambda = 2L \\
\text{One half-wavelength}
\]

**Topic: Uncertainty Principle**

- Heisenberg Uncertainty principle
- Arises from wave nature of particles.
- Precise position & momentum cannot be measured at the same time.
- Highly accurate momentum (wavelength) means position is uncertain
- Can localize particle by superimposing many wavelengths, so momentum is uncertain.
- Quantum mechanical tunneling.

**Quantum-mechanical tunneling**

**Additional Lecture Material**

- Spin
  - An additional quantum property of a particle
- Indistinguishability and symmetry
  - Fermions and Bosons
  - Pauli exclusion principle
- Physics of solids
  - Energy bands in a solid
  - Metals, insulators, and semiconductors
  - Superconductors

**Topic: spin**

- Free electron, by itself in space, not only has a charge, but also acts like a bar magnet with a N and S pole.
- Since electron has charge, could explain this if the electron is spinning.
- Then resulting current loops would produce magnetic field just like a bar magnet.
- But...
  - Electron is NOT spinning.
  - As far as we know, electron is a point particle.
Nucleus and radioactivity

- **Structure of the nucleus**
  - Nucleus has small size, large energy scale
  - Strong force holds nucleus together
  - Isotope: different neutron #, same proton #
  - Nuclear binding energy different for different nuclei
- **Radioactive decay**
  - ‘Unstable’ nuclei decay by emitting radiation
  - Alpha, beta, gamma decay
  - Decay half-life and carbon-dating
    - Decay is a random process
    - Half-life characterizes decay rate

Topic: Structure of the nucleus

- Nucleus is small, tightly bound system of protons & neutrons.
- Proton number determines the element.
- Different isotopes have different # neutrons.
- Some isotopes unstable, radioactively decay
- Nucleus held together by the strong nuclear force
  - Stronger than coulomb force,
  - But much shorter range than coulomb force.
- Strong force actually between quarks, internal constituents of the neutron/proton.
  ‘Leaks’ out to appear as an attractive force.

Size & structure of nucleus

- Nucleus consists of protons and neutrons densely combined in a small space (~10^{-14} m)
  - Protons have a positive electrical charge
  - Neutrons have zero electrical charge (are neutral)
- Spacing between these nucleons is ~ 10^{-15} m
- Size of electron orbit is 5x10^{-11} m
- Nucleus is 5,000 times smaller than the atom!

Isotopes

- Both ^{12}\text{C} and ^{14}\text{C} have same chemical properties.
- This is why they are both called carbon. Same # electrons and hence same # protons in nucleus.
- But the nuclei are different. They have different number of neutrons. These are called isotopes.
- Difference is most easily seen in the binding energy.
- Nuclei that are bound more tightly are less likely to ‘fall apart’.
- In fact ^{14}\text{C} is radioactive. It is unstable to emission of an electron.

Nuclear binding energy

These pulled together by strong force

Requires lots of work to pull them apart

So energy of nucleus is LESS than that of isolated nucleons...
...and energy is released when nucleons bind together.

Topic: Radioactive Decay

- Some nuclei spontaneously emit radiation: alpha, beta, gamma
- Radioactive half-life
- Decay type understood in terms of number neutrons, protons.
- Understand in terms of weak interaction, Quark internal structure.
**Alpha and beta decay**

- **Alpha decay:** chunk of nucleus ejected
- **Beta decay:** electron emitted
  - Neutron changes to proton

**Beta+ and gamma decay**

- Nucleus emits positron.
  - Proton changes to neutron
- Nucleus changes quantum state from high energy to low energy

**Topic: Radioactive half-life**

- Example of random decay.
- Start with 8 identical radioactive nuclei
- Suppose probability of decaying in one second is 50%.

**Carbon Dating**

- $^{14}$C has a half-life of ~6,000 years, continually decaying back into $^{14}$N.
- Steady-state achieved in atmosphere, with $^{14}$C/$^{12}$C ratio of 1:1 trillion (1 part in 10^{12})

- As long as biological material alive, atmospheric carbon mix ingested (as CO$_2$), ratio stays fixed.
- After death, no exchange with atmosphere. Ratio starts to change as $^{14}$C decays