**Hour Exam 2**: Wednesday, Oct. 27

- In-class (1300 Sterling Hall)
- Twenty multiple-choice questions
- Will cover: 8.1-8.6 (Light and E&M)
  9.1-9.5 (E&M waves and color)
  10, 11 (Relativity)

- You should bring
  - Your student ID
  - 1 page notes, written double sided
  - Calculator
  - Pencil for marking answer sheet
From Last Time...

- World line: path of particle through space-time.
- World line is made up of sequence of events, plotted as points in space-time.
- Different inertial reference frames represented as ‘tilted’ coordinate axes on space-time diagram.
- Event has different coordinates (space & time) measured on these coordinate axes.
- But the combination $x^2-c^2t^2$ is ‘universal’ in that it is measured to be the same for all observers.
- Suggests that space & time are ‘components’ of space-time, a conceptually new quantity.
Today: Forces, Work, and Energy in Relativity — or, What about Newton’s laws?

• Relativity dramatically altered our perspective.
  - But clearly objects still move, spaceships are accelerated by thrust, work is done, energy is converted.

• All the things we studied in the first six weeks.

• But some things are different in relativity.
\[ V_{ball} = \frac{v_{ball} + v_{motorcycle}}{1 + \frac{v_{ball}v_{motorcycle}}{c^2}} \]

- As motorcycle velocity approaches \( c \), \( V_{ball} \) also gets closer and closer to \( c \)
- End result: nothing exceeds the speed of light.

\[ v_{ball} = 0.96c \]
Newton again

- Fundamental relations of Newtonian physics
  - $acceleration = \frac{Force}{mass}$
  - $acceleration = \frac{\text{change in velocity}}{\text{change in time}}$
  - $Work = Force \times distance$
  - $Kinetic\ Energy = \left(\frac{1}{2}\right) (mass) \times (velocity)^2$
  - $Change\ in\ Kinetic\ Energy = net\ work\ done$

- Newton predicts that a constant force gives
  - Constant acceleration
  - Velocity proportional to time
  - Kinetic energy proportional to $(velocity)^2$
Momentum

- Newton’s laws become simpler when we use the **momentum** of a particle.
- \( \text{Momentum} = p = (\text{mass}) \times (\text{velocity}) = mv \)
- Then
  \[
  \text{Force} = \text{mass} \times \frac{\text{change in velocity}}{\text{change in time}}
  \]
  becomes
  \[
  \text{Force} = \frac{\text{change in momentum}}{\text{change in time}}
  \]
- Mass and velocity have been merged into a new quantity.
- For a variety of reasons, momentum is much more central to physics than is velocity.
Applying a constant force

- In the common case of a particle initially at rest, then subject to a constant force starting at $t=0$, we can write

\[ F = \frac{p}{t} \]

Giving $p = Ft = (\text{Force}) \times (\text{time})$

Momentum increases without bound as time increases
Forces in relativity

- Newton says a constant force produces a constant acceleration
- Velocity increases without bound.

Relativity says *no*.
The effect of the force gets smaller and smaller as velocity approaches speed of light.
Relativistic speed of particle subject to constant force

- At small velocities (short times) the motion is described by Newtonian physics.
- At higher velocities, there are substantial deviations.
- The velocity never exceeds the speed of light.

\[ \frac{v}{c} = \frac{t/t_o}{\sqrt{(t/t_o)^2 + 1}}, \quad t_o = \frac{F}{m_o c} \]
How does this come about?

- Momentum is such a fundamental and useful quantity in physics that the statements

\[
\text{Momentum is constant for zero force}
\]

and

\[
\frac{\text{change in momentum}}{\text{change in time}} = \text{Force}
\]

should remain true in some form.
Relativistic momentum

• Relativity concludes that the Newtonian definition of momentum \((p_{\text{Newton}} = mv = \text{mass} \times \text{velocity})\) is accurate at low velocities, but not at high velocities.

• The relativistic momentum is

\[
p_{\text{relativistic}} = \gamma mv
\]

\[
\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}
\]

Relativistic gamma
mass
velocity
Was Newton wrong?

• Have just shown that relativity requires a different concept of momentum

\[ p_{\text{relativistic}} = \gamma mv \]
\[ \gamma = \frac{1}{\sqrt{1-(v/c)^2}} \]

• But this is not really so different.
• For small velocities (much less than speed of light \( c \)) \( \gamma \approx 1 \), and so \( p_{\text{relativistic}} \approx mv \)
• This is Newton’s momentum.
• Differences only occur at velocities that are a substantial fraction of the speed of light.
Relativistic Momentum

- Momentum can be increased arbitrarily, but velocity never exceeds $c$
- We still use $\frac{\text{change in momentum}}{\text{change in time}} = \text{Force}$ so for constant force we still have momentum $= \text{Force} \times \text{time}$, but the velocity never exceeds $c$
- Momentum has been redefined.

$$P_{\text{relativistic}} = \gamma mv = \frac{mv}{\sqrt{1 - (v/c)^2}}$$

Relativistic momentum for different speeds.
How can we understand this?

- This result says that at high speeds the acceleration \( \left( \frac{\text{change in velocity}}{\text{change in time}} \right) \) is much smaller than at low speeds, for the same force.

- From Newtonian perspective, the relation between Force and acceleration is determined by the mass.

- We could say that the mass increases as the speed increases.

\[
p_{\text{relativistic}} = \gamma mv = (\gamma m)v \equiv m_{\text{relativistic}}v
\]

- To be clear about this, it is written

\[
p_{\text{relativistic}} = \gamma m_o v = (\gamma m_o)v \equiv mv
\]

- \( m_o \) is called the rest mass.
- mass \( m \) now depends on vel.

\[
\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}, \quad m = \gamma m_o
\]
Relativistic mass

- Could say that particle becomes extremely massive as speed increases ($m = \gamma m_0$)

- But could also say that relativistic momentum has new form ($p = \gamma m_0 v$)

- Mainly semantics — just an interpretation of relativity.
Example

• An object moving at half the speed of light relative to a particular observer has a rest mass of 1 kg. What is it’s mass measured by the observer mentioned above?

\[
\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.5c/c)^2}} = \frac{1}{\sqrt{1-0.25}}
\]

\[
= \frac{1}{\sqrt{0.75}} = 1.15
\]

So measured mass is 1.15 kg
Work and Energy

• Newton says that Work = Force x Distance, and that net work done on an object changes the kinetic energy.

• Used this to find the classical kinetic energy

\[ KE_{Newton} = \frac{1}{2} mv^2 \]

• Since \( p = mv \) according to Newton, this is also

\[ KE_{Newton} = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \]
Relativistic Kinetic Energy

- Might expect this to change in relativity, since the change in velocity due to a force is different than in Newtonian physics.

- Can do the same analysis as we did with Newtonian motion to find

\[ KE_{\text{relativistic}} = (\gamma - 1)m_o c^2 \]

- Doesn’t seem to resemble Newton’s result at all.

- However for small velocities, it does reduce to the Newtonian form

\[ KE_{\text{relativistic}} \approx \frac{1}{2} m_o v^2 \text{ for } v << c \]
Relativistic Kinetic Energy

- Can see this graphically as with the other relativistic quantities.
- Kinetic energy gets arbitrarily large as speed approaches speed of light.
- Is the same as Newtonian kinetic energy for small speeds.
Total Relativistic Energy

• The relativistic kinetic energy is

\[ KE_{\text{relativistic}} = (\gamma - 1)m_o c^2 \]

\[ = \gamma m_o c^2 - m_o c^2 \]

- Depends on velocity
- Constant, independent of velocity

• Write this as

\[ \gamma m_o c^2 = KE_{\text{relativistic}} + m_o c^2 \]

- Total energy
- Kinetic energy
- Rest energy
Mass-energy equivalence

• This results in Einstein’s famous relation

\[ E = \gamma m_o c^2, \text{ or } E = mc^2 \]

• This says that the total energy of a particle is related to its mass.

• Even when the particle is not moving it has energy.

• We could also say that mass is another form of energy
  - Just as the text talks of chemical energy, gravitational energy, etc, we can talk of mass energy.
Example

- In a frame where the particle is at rest, its total energy is $E = m_o c^2$
- Just as we can convert electrical energy to mechanical energy, it is possible to tap mass energy.
- A 1 kg mass has $(1\text{kg})(3\times10^8\text{m/s})^2=9\times10^{16} \text{ J}$ of energy.
  - We could power 30 million 100 W light bulbs for one year!
  (~30 million sec in 1 yr)
Energy and momentum

- Relativistic energy is \[ E = \gamma m_0 c^2 \]
- Since \( \gamma \) depends on velocity, the energy is measured to be different by different observers
- Momentum also different for different observers
  - Can think of these as analogous to space and time, which individually are measured to be different by different observers.
- But there is something that is the same for all observers:
  \[ E^2 - c^2 p^2 = \left( m_0 c^2 \right)^2 \]
  = Square of rest energy
- Compare this to our space-time invariant \[ x^2 - c^2 t^2 \]
A relativistic perspective

• Suggests that the concepts of space, time, momentum, energy that were useful to us at low speeds for Newtonian dynamics prove to be a little confusing near light speed.

• Relativity needs new conceptual quantities, such as space-time and energy-momentum which are a little foreign to us.

• Trying to make sense of relativity using space and time separately leads to effects such as time dilation and length contraction.

• In the mathematical treatment of relativity, just such space-time and energy-momentum objects are used.