From Last Time...

- All objects show both wave-like properties and particle-like properties.

- Electromagnetic radiation (e.g. light) shows interference effects (wave-like properties), but also comes in discrete photons of energy \( hf \) (particle-like properties).

- Matter clearly shows particle-like properties, but also shows interference and diffraction effects (wave-like properties).
Electron diffraction

- Wavelength determined by electron momentum

\[
\text{Wavelength} = \frac{\text{Planck's constant}}{\text{momentum}}
\]

- Momentum determined by electron energy

- Energy determined by accelerating voltage (energy in electron-volts (eV) = voltage)

\[
\text{Wavelength in } \text{nm} = \frac{1.23 \ eV^{1/2} - nm}{\sqrt{E_{\text{kinetic}}}}
\]

Energy in eV
Increasing the energy decreases the wavelength.

When electron wavelength comparable to differences in path length, destructive interference can occur.

Appears as spatial directions in which reflected electrons are never seen.

Wavelength in $nm = \frac{1.23 \ eV^{1/2} - nm}{\sqrt{E_{kinetic}}}$
Making a particle out of waves

440 Hz + 439 Hz

440 Hz + 439 Hz + 438 Hz

440 Hz + 439 Hz + 438 Hz + 437 Hz + 436 Hz
Using quantum mechanics

• Quantum mechanics makes astonishingly accurate predictions of the physical world

• Can apply to atoms, molecules, solids.

• An early success was in understanding
  - Structure of atoms
  - Interaction of electromagnetic radiation with atoms
Planetary model of atom

- Positive charge is concentrated in the center of the atom (nucleus).
- Atom has zero net charge:
  - Positive charge in nucleus cancels negative electron charges.
- Electrons orbit the nucleus like planets orbit the sun.
- (Attractive) Coulomb force plays role of gravity.
Planetary model

- Bohr also retained the particle picture.
- Since electron is in a circular orbit, it’s velocity is continually changing direction (centripetal acceleration = $v^2/r$)
- The source of this acceleration is the Coulomb force between the positive nucleus and the negative electron $ke^2/r^2$

\[
\text{acceleration} = \frac{v^2}{r} = \frac{\text{Force}}{\text{mass}} = k \frac{e^2}{mr^2}
\]
Difference between atoms

• No net charge to atom
  - number of orbiting negative electrons same as number of positive protons in nucleus
  - Different elements have different number of orbiting electrons

• Hydrogen: 1 electron
• Helium: 2 electrons
• Copper: 29 electrons
• Uranium: 92 electrons!
• Organized into periodic table of elements
Elements in same column have similar chemical properties.
Problems with planetary model

- No clear predictions of size of electron orbits
- No explanation of why different elements have different chemical properties.
- Has particular problems with predictions for emission of radiation by atoms.
Line spectra from atoms

- Atoms do emit radiation, but only at certain discrete frequencies.
- Frequencies are unique for different atoms.
- Spectrum is an atomic ‘fingerprint’, used to identify atoms (e.g. in space).

![Diagram showing line spectra for Hydrogen and Mercury](image)
Planetary model and radiation

- Circular motion of orbiting electrons causes them to emit electromagnetic radiation with frequency equal to orbital frequency.

- Same mechanism by which radio waves are emitted by electrons in a radio transmitting antenna.

- In an atom, the emitted electromagnetic wave carries away energy from the electron.
  - Electron predicted to continually lose energy.
  - The electron would eventually spiral into the nucleus
  - *However most atoms are stable!*
Hydrogen emission spectrum

- Hydrogen is simplest atom
  - One electron orbiting around one proton.

- The Balmer Series of emission lines empirically given by
  \[
  \frac{1}{\lambda_m} = R_H \left( \frac{1}{2^2} - \frac{1}{n^2} \right)
  \]

  \(n = 4, \lambda = 486.1 \text{ nm}\)

  \(n = 3, \lambda = 656.3 \text{ nm}\)
Hydrogen emission

• This says hydrogen emits only photons of a particular wavelength, frequency

• Photon energy = $hf$, so this means a particular energy.

• Conservation of energy:
  - Energy carried away by photon is lost by the orbiting electron.
The Bohr atom

- Retained ‘planetary’ picture
- Only certain orbits are stable
- Radiation emitted only when electron jumps from one stable orbit to another.
- Here, the emitted photon has an energy of \( E_{\text{initial}} - E_{\text{final}} \)
Energy conservation for Bohr atom

- Each orbit has a specific energy 
  \[ E_n = -\frac{13.6}{n^2} \]
- Photon emitted when electron jumps from high energy to low energy orbit.
  \[ E_i - E_f = h f \]
- Photon absorption induces electron jump from low to high energy orbit.
  \[ E_f - E_i = h f \]
- Agrees with experiment!
Example: the Balmer series

- All transitions terminate at the \( n=2 \) level
- Each energy level has energy \( E_n = -\frac{13.6}{n^2} \) eV
- E.g. \( n=3 \) to \( n=2 \) transition
  - Emitted photon has energy
    \[
    E_{\text{photon}} = \left( -\frac{13.6}{3^2} \right) - \left( -\frac{13.6}{2^2} \right) = 1.89 \text{ eV}
    \]
  - Emitted wavelength
    \[
    E_{\text{photon}} = hf = \frac{hc}{\lambda}, \quad \lambda = \frac{hc}{E_{\text{photon}}} = \frac{1240 \text{ eV} - nm}{1.89 \text{ eV}} = 656 \text{ nm}
    \]
Spectral Question

Compare the wavelength of a photon produced from a transition from $n=3$ to $n=1$ with that of a photon produced from a transition $n=2$ to $n=1$.

A. $\lambda_{31} < \lambda_{21}$

B. $\lambda_{31} = \lambda_{21}$

C. $\lambda_{31} > \lambda_{21}$

$E_{31} > E_{21}$ so $\lambda_{31} < \lambda_{21}$
But why?

• Why should only certain orbits be stable?

• Bohr had a complicated argument based on “correspondence principle”
  - That quantum mechanics must agree with classical results when appropriate (high energies, large sizes)

• But incorporating wave nature of electron gives a natural understanding of these ‘quantized orbits’
Electron waves in an atom

- Electron is a wave.
- In the orbital picture, its propagation direction is around the circumference of the orbit.
- Wavelength = $h / p$ ($p=$momentum, and energy determined by momentum)
- How can we think about waves on a circle?
Waves on a circle

- Here is my ‘ToneNut’
- Like a flute, but in the shape of a doughnut.
- Produces tone at particular pitch.
- Air inside must be vibrating at that frequency (red line)
- Sound wave inside has corresponding wavelength \( \lambda = \nu / f \)
- What determines the frequency/wavelength of the sound?
Standing waves

- Easier to think about in a normal wind instrument, or vibrations of a string.
- Wind instrument with particular fingering plays a particular pitch, particular wavelength.
- Guitar string vibrates at frequency, wavelength determined by string length.

\[ \lambda = \frac{L}{2} \]

\[ f = \frac{v}{\lambda} \]
Standing Waves

- When a traveling wave reflects back on itself, it creates traveling waves in both directions.
- The wave and its reflection interfere according to the superposition principle.
- With exactly the right frequency, the wave will appear to stand still.
  - This is called a *standing wave*.
Standing Waves on a String

- Zeros of the wave must occur at the ends of the string because these points are fixed.
- Zeroes occur every half-wavelength.
- Zeroes are called ‘nodes’ of the wave.
Standing waves: wind instrument

- Ends are open, ‘antinodes’ at ends.
- Integer number of half-wavelengths fit inside the instrument.

\[
\begin{align*}
\lambda_1 &= \frac{2L}{2} = 2L \\
\frac{f_1}{f} &= \frac{v}{\lambda} = \frac{v}{2L} \\
\lambda_2 &= \frac{L}{2} \\
\frac{f_2}{f} &= \frac{v}{\lambda} = 2f_1 \\
\lambda_3 &= \frac{L}{3} \\
\frac{f_3}{f} &= \frac{3v}{2L} = 3f_1
\end{align*}
\]

(a) Open at both ends
Standing Waves on a String, cont.

- Only possible waves are standing waves.
- The frequency of first possible standing wave is called the **fundamental frequency**
- All possible standing wave frequencies are an integer multiple of the fundamental frequency.
- Called **harmonics**

\[ \lambda = \frac{2L}{2} \]

\[ \lambda = 2L \]

\[ \lambda = \frac{3L}{2} \]

*Integer number of half-wavelengths fits in the space.*
Why not other wavelengths?

- Waves in the string that are not in the harmonic series are quickly damped out
  - In effect, when the string is disturbed, it “selects” the standing wave frequencies
- Reflection from ‘end’ interferes destructively and ‘cancels out’ wave.
- Same happens in a wind instrument...
  ... and in an atom!
Waves on a ring

Wavelength

- Condition on a ring slightly different.
- Integer number of wavelengths required around circumference.
- Otherwise destructive interference occurs when wave travels around ring and interferes with itself.
Electron standing-waves on an atom

- Electron wave extends around circumference of orbit.
- Only integer number of wavelengths around orbit allowed.
Electron Standing Waves

- This gives a condition on the electron wave.
- Integer number of deBroglie wavelengths must fit on circumference of the orbit.
- Circumference = \((2\pi) \times \text{(orbit radius)} = 2\pi r\)
- So condition is \(2\pi r = n\lambda = n\frac{h}{p} = n\frac{h}{mv}\)
- Momentum \(p = (\text{mass}) \times (\text{velocity}) = mv\)

Thus wave considerations give a relation between orbit radius and electron momentum.
Particle properties: electron is still orbiting!

- Bohr also retained the particle picture.
- Since electron is in a circular orbit, it’s velocity is continually changing direction (centripetal acceleration $= \frac{v^2}{r} = \frac{\text{Force}}{\text{mass}}$)
- The force causing this acceleration is the Coulomb force $\frac{ke^2}{r^2}$ between the positive nucleus and the negative electron

Thus particle considerations also give a relation between orbit radius and electron momentum.
Toward quantized orbits

- Electron standing wave: 
  \[ 2\pi r = n\lambda = n\frac{h}{p} \]
  
  - Orbit circumference
  - Integer # of deBroglie wavelengths

  Pronounced h-bar, \( \hbar = \frac{h}{2\pi} \)

- Electron ‘planetary’ orbit:
  \[ \frac{v^2}{r} = k \frac{e^2}{mr^2} \]
  
  - centripetal acceleration
  - Coulomb force / mass

  \[ p^2 = \frac{mke^2}{r} \]
Only certain radius orbits permitted by quantum mechanics

Equate wave result with particle result — orbital radius is quantized

Permitted radii:

\[ r_n = n^2 \frac{\hbar^2}{mke^2} = n^2 a_o \]

\[ a_o = 0.529\text{Å} = \text{Bohr radius} \]

\[ r_3 = 9a_o = 4.76\text{Å} \]
\[ r_4 = 16a_o = 8.46\text{Å} \]
Quantum States

- Orbit labeled by quantum number $n$
- Referred to as a quantum state
- Electron has different properties in each quantum state
  - Different orbital radius
  - Different wavelength (momentum)
  - Different energy
Energy of the quantum state

- Electron energy includes
  - Potential energy (from Coulomb interaction)
  - Kinetic energy (energy of motion, from momentum)
- Combining these contributions, Bohr found that allowed electron energies are quantized as

\[ E_n = -\frac{13.6}{n^2} \text{ eV} \]

- Just what is required to explain spectral lines