From last time...

- Quantum field theory is a relativistic quantum theory of fields and interactions.
- Fermions make up matter, and bosons mediate the forces by particle exchange.
- Lots of particles, lots of interactions, but can be unified to some extent.
- Electroweak accepted, leaving quarks and leptons as separate particles, strong and electroweak gauge bosons, Higgs boson.
- GUTs attempt to include the strong force.
Today

• What about gravity - where does it fit in?
• Classical vs quantum theories of gravity
• Classical unification of gravity & electromagnetism
• Extra dimensions!
• String theory
Gravity

• Haven’t talked recently about gravity.
• Gravity not particularly relevant at the scale of particle physics, because the particles are not massive enough to interact gravitationally.
• But shouldn’t we be able to explain gravity in the same breath as particles and interactions?
• Can’t we unify both quantum mechanics and gravity into a theory of everything?
Einstein’s gravity

- General Relativity is a classical theory.
- Einstein was a classical guy, even though he received Nobel for photoelectric effect, general theory of relativity has nothing to do with quantum mechanics.
- General relativity has to do with curved space-time, and motion of objects in that curved space time.
Kaluza-Klein: EM & gravity

• Connect electromagnetism and gravity in a classical relativistic theory.

• Kaluza and Klein found a theory in five dimensions (four space & one time) with one interaction (5-dimensional gravity).

• When one of the dimensions was ‘compactified’, two interactions resulted: gravity and electromagnetism.

• What appears to us as two distinct interactions originate from only one.
Extra dimensions?

• How can there be extra dimensions?

• Can imagine more physical dimensions, but we do not see them

• We would be unaware of them if they were very small, e.g. very strongly curved a la GR

• Example: A power cable from far away looks like a one-dimensional line.

• Up close we see that it is really a 2D surface
  - The 2nd dimension was curved so much we do not notice it.
Compactification in Kaluza-Klein

- The process of ‘rolling up’ the extra dimension to leave four space-time dimensions...
- ...made the 5-dimensional geometrical gravitational interaction appear as two different interactions in 4D:

| Electromagnetism | — | Gravity |

Another unification!
QFT and GR don’t mix

- GR leads to gravitational waves.
- These are classical waves that should appear as particles in a quantum field theory.
- But “quantizing” GR gives untamable infinities
- **Interactions** in QFT are point-like
• The Kaluza-Klein theory cannot be quantized.

• When standard methods are applied, the theory gives answers that have no physical meaning.

• E.g., infinite values for energies.

• Physicists say the theory is not renormalizable.

• This can be traced back to the graviton being a spin-2 particle. (classical analog is polarization of gravity wave).

• All the other exchange bosons are spin 1.
Supersymmetry: ‘solution’ to a different problem

- Matter (fermions) and forces (bosons) behave differently.
- Half integer spin (1/2) are fermions and integer spin (1) are bosons.
- Quantum version of “rotation”, which is connected to rotational symmetry transformations.
- Space-time symmetries can be uniquely extended with supersymmetries that connect fermions and bosons.

Fermions

Bosons
Supersymmetry (SuSy)

Every fermion has a boson partner and vice versa: superpartners (compare to anti-particles)
String theory

• A string is a fundamental quantum mechanical object that has a small but nonzero spatial extent.

• Just like a particle has a mass, a string has a ‘tension’ that characterizes its behavior.

• Quantum mechanical vibrations of the string correspond to the particles we observe.
The idea of String Theory is that this diversity of quarks and leptons comes from one String.
And the forces too!

all forces

& all matter

into one object...

...a **String**!

Matter

$e^-, p^+, n$

Force

photon, gluon, graviton

Strong, Weak,
Gravitation,
Electromagnetism
electrons, protons, neutrons...
• Strings can vibrate in different ways

For example:

**Guitar string**

Different vibration  Different sound!

• **Fundamental** string

Different vibration  Different particles!

- electron
- photon
- graviton

Not exactly!
What are these strings?

We describe them only in terms of a fundamental tension – as for a rubber band

\[ \lambda \]

How big are they?

A particle of energy \( E \) has a wavelength

\[ E = \frac{\hbar c}{\lambda} \]

So can probe down to scales of order \( \lambda \). . . . So far we’re down to 1/1000\(^{th}\) size of atomic nucleus . . . strings could be \( 10^{19} \) times smaller!
What about QM?

How can we ‘quantize’ the theory?

We quantized the planetary model of the hydrogen atom by saying the electron was a wave.

A consequence of this was the uncertainty relation between momentum and position.

A string is a continuous length of spatial positions, each of which can be moving with some momentum.

Quantize it by requiring each pair \([x,p(x)]\) to obey the uncertainty relation.
Some problems

• Strings are collections of points
  – an infinite number of points

• This can make for very complex behavior.

• Theory for a classical relativistic string worked

• But quantizing the string leads to a physical theory only in 26 dimensions!

• Rather large, but at least the number of dimensions of space-time is an output of the theory rather than an input.
Results of the theory

• The first string excitation is a particle with *imaginary* mass — a tachyon (negative mass squared = negative energy)

• Clearly off-base.

• But the next excitation is a massless spin-2 particle satisfying general relativity
  - The graviton!

• So string theory became a theory of gravity
Superstrings

• Imposing supersymmetry on strings gets rid of the tachyon - it is no longer a solution.

• Additionally, the number of dimensions required for consistency drops from 26 to 10!

• So far so good.

• Fundamental object is now a ‘superstring’
  - A string with extra degrees of freedom that make it supersymmetric
Types of strings

There are two basic types of strings: open and closed.

But the natural interactions of strings is via their endpoints - strings join together when their endpoints touch.

Then open strings can be come closed strings.
String Interactions

- Strings interact by joining and splitting
- Solves point interaction problem of gravitation in QFT
Five theories of strings

• There are at least five originally distinct theories of superstrings.
• They are differentiated by the details of string behavior.
  - One ‘open string’ theory (Type I)
  - Two distinct ‘closed string’ theories (Types IIA & B)
  - Two heterotic string theories (combo of supersymmetric / non-supersymmetric strings)
• Now known that these are ‘equivalent’ in the sense that they are special cases of a larger (unknown) ‘M-theory’
Extra dimensions in string theory

• Superstring theory requires a 10 dimensional spacetime, to eliminate quantum states with unphysical negative probabilities (ghosts).

• How do we get from 10 dimensions down to 4?

• Two main proposals:
  - Roll up the extra dimensions into some very tiny space of their own. **Kaluza-Klein** compactification.
  - Make the extra dimensions really big, but constrain all the matter and gravity to propagate in a 3D subspace called the three-brane. These types of theories are called braneworlds.
Compactification

• Compactification requires rolling up the extra six dimensions in a consistent way.

• The exact way to roll these up determines the ways in which the strings can vibrate.

• In particular compactifications, most of the matter and force particles we know about can be found.
Superstring Dimensions

- Since we can observe only 3 spatial and 1 time dimensions, the extra 6 dimensions (in a 10D string theory) are curled up to a very small size.
- The shape of the curled up dimensions is known mathematically as a Calbi-Yau space.

2D slice of 6D Calbi-Yau space
Superstring Universe

- At each point in 3D space, the extra dimensions exist in unobservably small Calabi-Yau shapes
Gravity is different to the other forces - it’s only attractive...

In General Relativity this shows up in that gravitational waves have different polarizations to electromagnetic waves
Bosons in string theory

String theory unifies gravity and other forces

String theory is a quantum theory of gravity. Is it the quantum theory of gravity? - entering realm of speculation!
Back to the ends

- Different ‘boundary conditions’ for the ends of the strings.
- Might consider them free to move throughout space.
- But they could also be anchored to some point if they have an open end.
- Graviton cannot be anchored - it is an excitation of a closed string (with no ends).
Membranes too...

Recently it has been discovered that open strings may be restricted to a sub-space or membrane.

Electromagnetic particles live on a sub-space or "brane".

Gravitons live in a higher dimension "bulk".

"Existence proof" for such a world.
Could Our Universe Be A Brane?

The strength of gravity is determined by the number of spatial dimensions (gravitons spread out around mass)

\[ F = \frac{GMm}{r^2} \quad D=3+1 \]

But... we don’t know anything about gravity on length scales below 1mm...
R could = 1mm... and we wouldn’t know it!

If so we’ve miscalculated the strength of gravity (G) - it could become strong in our particle accelerators at any new energy!!
Other Branes

Why should there be only one brane in the higher dimension spacetime?

There could be entire Universes only mm away!

The matter on the other brane will only interact with our world gravitationally - it’s dark matter....

Until we can produce high energy gravitons that are strongly interacting there’s little way to directly probe this idea though
Brane Collisions

There’s no reason branes should be static in the extra dimensions.... So they could collide!

This would be catastrophic!

Huge amounts of energy would be dumped into our Universe....

Could that have triggered the currently observed expansion of the Universe??!!
Strings on Nova

• **This Week on NOVA**

• "The Elegant Universe: Einstein's Dream" and "String's the Thing" airs on PBS Tuesday, December 21 from 8 to 10 p.m. Check local listings as dates and times vary.