From Last Time...

Newton’s three laws of motion:

1) Law of inertia
2) $F=ma$ (or $a=F/m$)
3) Action and reaction
   (forces always come in pairs)
Question

If an apple falls toward the Earth, why doesn’t the moon fall toward the Earth?

A. The moon is too big
B. The moon is too far away
C. The moon *does* fall toward the earth.
Velocity of the moon

What is the direction of the Velocity of the moon?
Acceleration = \frac{\text{change in velocity}}{\text{change in time}}

- Speed is same, but direction has changed
- Velocity has changed
How has the velocity changed?

Velocity at time $t_1$

Velocity at time $t_2$

Change in velocity

Centripetal acceleration = $v^2/r$, directed toward center of orbit. $r = \text{radius of orbit}$
Earth’s pull on the moon

- The moon continually accelerates toward the earth,
- But because of its orbital velocity, it continually misses the Earth.
- The orbital speed of the moon is constant, but the direction continually changes.
- Therefore the velocity changes with time.

True for any body in circular motion
Experiment

\[ F = m_2 g \]

Acceleration of ball \( m_1 = \frac{F}{m_1} = \frac{m_2 g}{m_1} \)

\( m_1 \) accelerates inward in response to force \( m_2 g \)

Acceleration = \( \frac{v^2}{r} \) for circular motion
Newton’s falling moon

Throwing the ball fast enough results in orbital motion

From Newton’s Principia, 1615
Question

A 2 newton apple falls from a tree. What is the net force on the apple?

2 newtons downward

What is the acceleration?

g~10 m/s² downward
Question, part 2

I throw the 2 N apple horizontally. After I throw it, what is the net force on the apple?

2N downward

What is the acceleration of the apple?

$g \approx 10 \text{ m/s}^2$ directly downward

This is because, after I throw the apple, gravity is the only force. The only acceleration is due to gravity.
After the dart leaves the gun, the only force is from gravity. The only deviation from straight-line motion is an acceleration directly downward. This is the same as the monkey.
Acceleration of moon

• So the moon is accelerating at \( \frac{v^2}{r} \) \( m/s^2 \)

  directly toward the earth!

• This acceleration is due to the force of gravity.

• Is this equal to \( g \), the acceleration of the apple?
  - Can calculate it directly from moon’s orbital speed, and the Earth-moon distance.
The radius of the earth

- “Originally” from study of shadows at different latitudes by Eratosthenes!
- \( R(\text{earth}) = 6500 \text{ km} \)
Distance and diam. of moon

• The diameter of the moon is the diameter of its shadow during a solar eclipse. From the diameter d and angular size d/r~5 deg, infer distance r~60*r(earth).
Moon acceleration, cont

• Distance to moon = 60 earth radii \( \sim 3.84 \times 10^8 \) m

• Speed of moon?
  
  Circumference of circular orbit = \( 2\pi r \)

\[
\text{Speed} = \frac{\text{orbital distance}}{\text{orbital time}} = \frac{2\pi r}{27.3 \text{ days}} = 1023 \ m/s
\]

Centripetal acceleration = 0.00272 \( m/s^2 \)

This is the acceleration of the moon due to the gravitational force of the Earth.
Distance dependence of Gravity

• The gravitational force depends on distance.
• Moon acceleration is

$$\frac{9.81 \text{ m/s}^2}{0.00272 \text{ m/s}^2} \approx 3600 \text{ times smaller than the acceleration of gravity on the Earth’s surface.}$$

• The moon is 60 times farther away, and $3600 = 60^2$
• So then the gravitational force drops as the distance squared

Newton: *I thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly.*
Equation for force of gravity

$$F_{\text{gravity}} \propto \frac{(\text{Mass of object 1}) \times (\text{Mass of object 2})}{\text{square of distance between them}}$$

$$F \propto \frac{m_1 \times m_2}{d^2}$$

For masses in kilograms, and distance in meters,

$$F = 6.7 \times 10^{-11} \frac{m_1 \times m_2}{d^2}$$
Example

- Find the acceleration of an apple at the surface of the earth

Force on apple \( F_{apple} = 6.7 \times 10^{-11} \frac{m_{Earth} \times m_{apple}}{d^2} \)

This is also the force on the Earth by the apple!

\( d = \) distance between center of objects \( \sim \) radius of Earth

Acceleration of apple \( \frac{F_{apple}}{m_{apple}} = 6.7 \times 10^{-11} \frac{m_{Earth}}{d^2} \)

\[
= 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \times \frac{5.98 \times 10^{24} \text{ kg}}{(6.37 \times 10^6 \text{ m})^2} = 9.83 \text{ m/s}^2
\]
Gravitational force decreases with distance from Earth

Force on apple = \( F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_{\text{Earth}} \times m_{\text{apple}}}{d^2} \)

So moving farther from the Earth should reduce the force of gravity

- Typical airplane cruises at ~5 mi = 8000 m
  - \( d \) increases from 6,370,000 m to 6,378,000 m
  - Not noticeable!
• International space station orbits at 350 km = 350,000 m
• \( d = 6,370,000 \text{ m} + 350,000 \text{ m} = 6,720,000 \text{ m} \)
• Again \( d \) has changed only a little, so that \( g \) is decreased by only \( \sim 2\% \).
So why is everyone floating around?

Edward M. (Mike) Fincke, Expedition 9 NASA ISS science officer and flight engineer, is pictured near freely floating freely in the Zvezda Service Module of the International Space Station. (NASA)

James S. Voss, Expedition Two flight engineer, looks over an atlas in the Zvezda Service Module. (NASA)
The space station is falling...

...just like Newton’s apple

- In its circular orbit, once around the Earth every 90 minutes, it is continuously accelerating toward the Earth at \(~9.8 \text{ m/s}^2\).
- Everything inside it is also accelerating at that same rate.
- The astronauts are freely falling inside a freely-falling ‘elevator’.
Supreme Scream – 300 feet of pure adrenaline rush

A freefall ride

\[ d = \frac{1}{2} at^2 \]

\[ t = \sqrt{\frac{2d}{a}} \]

\[ = \sqrt{\frac{2 \times 300 \text{ ft}}{32 \text{ ft/s}^2}} \]

\[ = 4.3 \text{ sec of freefall} \]
A little longer ride

Parabolic path of freely falling object
But gravitational force is only 2% weaker - Need to move farther from the Earth.
Question

Halfway to the moon, what is the acceleration of an apple due to the Earth’s gravity?

A. \( \frac{g}{2} \)
B. \( \frac{g}{4} \)
C. \( \frac{g}{900} \)

Moon is 60 Earth radii from the Earth. Halfway is 30 Earth radii. So apple is 30 times farther than when on surface. Gravitational force is \( (30)^2 \) times smaller = \( \frac{g}{900} \)
Acceleration of gravity on moon

- On the moon, an apple feels gravitational force from the moon.
- Earth is too far away.

\[
F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_{\text{moon}} \times m_{\text{apple}}}{r_{\text{moon}}^2}
\]

\[
\text{Accel. of apple on moon} = \frac{F_{\text{apple}}}{m_{\text{apple}}} = 6.7 \times 10^{-11} \frac{m_{\text{moon}}}{r_{\text{moon}}^2}
\]

Compare to accel on Earth = \(6.7 \times 10^{-11} \frac{m_{\text{Earth}}}{r_{\text{Earth}}^2}\)

\[
\frac{\text{accel. on moon}}{\text{accel. on Earth}} = \frac{m_{\text{moon}} / m_{\text{Earth}}}{(r_{\text{moon}} / r_{\text{Earth}})^2}
\]
Accel. of gravity on moon

\[
\frac{\text{accel. on moon}}{\text{accel. on Earth}} = \frac{\frac{m_{\text{moon}}}{m_{\text{Earth}}}}{\left(\frac{r_{\text{moon}}}{r_{\text{Earth}}}\right)^2}
\]

\[
= \frac{7.4 \times 10^{22} \text{ kg} / 6.0 \times 10^{24} \text{ kg}}{(1.7 \times 10^6 \text{ m} / 6.4 \times 10^6 \text{ m})^2}
\]

\[
= \frac{0.0123}{(0.265)^2} = 0.175 \approx \frac{1}{6}
\]
Gravitational force between small objects
Gravitatity force at large distances:
Stars orbiting our black hole

• At the center of our galaxy is a collection of stars found to be in motion about an invisible object.
Orbits obey Newton’s gravity, orbiting around some central mass

http://www.mpe.mpg.de/www_ir/GC/intro.html
What is the central mass?

- One star swings by the hole at a minimum distance $b$ of 17 light hours (120 A.U. or close to three times the distance to Pluto) at speed $v = 5000$ km/s, period 15 years.
- From the orbit we can derive the mass.
- The mass is 2.6 million solar masses.
- It is mostly likely a black hole!