From last time...

- Galilean Relativity
  - Laws of mechanics identical in all inertial ref. frames
- Einstein’s Relativity
  - All laws of physics identical in inertial ref. frames
  - Speed of light-c in all inertial ref. frames

One consequence
- Simultaneity: events simultaneous in one frame will not be simultaneous in another.

Concept test
An elevator freely falling in the Earth's gravitational field is NOT an inertial reference frame because

A. It is not moving at zero velocity.
B. It is not moving at constant velocity.
C. It is not moving with constant acceleration

Time and simultaneity

• What does it mean for two things to happen at the same time?

I do not define time, space, place, and motion, as being well known to all. — Newton

If you do not ask me what is time, I know it.
When you ask me, I cannot tell it. — St. Augustine

Simultaneity and relativity

• When are two events simultaneous?
  - Easy if they are at the same spatial location.
  - Different spatial locations require some synchronization, which requires signal transmission between locations.

Analogy with sound

• Suppose you hear two loud shots about 1/2 second apart.
• Did they occur at the same time?
  - Obviously not.
• But suppose you find out one of the shots was fired closer to you than the other.
• Sound travels at 340 m/s.
• The sound pulse closer to you would arrive first, even if they were fired at the same time.

But everyone can agree

• If you know your distance from the shots, you can easily determine if they were simultaneous.
• And everyone will agree with you, after doing the same correction for distance.
• You might even come up with a definition
  - Event $x_1$, $t_1$ is simultaneous with event $x_2$, $t_2$ if sound pulses emitted at $t_1$ from $x_1$ and at $t_2$ from $x_2$ arrive simultaneously at the midpoint between $x_1$ and $x_2$. 
**Relativistic simultaneity**

- Einstein came up with a similar definition for relativistic simultaneity.

- Simultaneity of separated events:
  - Event \((x, t)\) is simultaneous with event \((x', t')\) if light signals emitted at \(t\) from \(x\) and at \(t'\) from \(x'\) arrive simultaneously at the midpoint between \(x\) and \(x'\).

- **BUT**, due to the constancy of the speed of light we are forced into the conclusion that not everyone will agree that two events are simultaneous (occurred at the same time).

**Simultaneity and relativity, cont**

- Means there is no universal, or absolute time.
  - The time interval between events in one reference frame is generally different than the interval measured in a different frame.
  - Events measured to be simultaneous in one frame are in general not simultaneous in a second frame moving relative to the first.

**Simultaneity thought experiment**

- Boxcar moving with constant velocity \(v\) with respect to observer \(O\) on the ground.
- Observer \(O'\) rides in exact center of the boxcar.
- Two lightning bolts strike the ends of the boxcar, leaving marks on the boxcar and the ground underneath.
- Observer \(O\) on the ground finds that she is halfway between the scorch marks.

**Simultaneity, continued**

- Observer \(O\) (on the ground) also observes that light waves from each lightning strike at the boxcar ends reach her at exactly the same time.
- Since each light wave traveled at \(c\), and each traveled the same distance (since \(O\) is in the middle), the lightning strikes are simultaneous in the frame of the ground observer.

**The events in boxcar \((O')\) frame**

- When light from front flash reaches boxcar observer \(O'\), he has moved away from rear flash.
  - Light from rear flash has not yet reached him.
- Both light waves travel at \(c\) in the boxcar frame, therefore the lightning strikes at the boxcar ends are **NOT simultaneous in the boxcar frame**.

**Sound waves**

- If we were using sound pulses (the crack from the lightning hitting the train), there would be no problem.
- The air (medium that transports sound waves) provides an absolute reference frame.
- Speed of sound is \(340\) m/s relative to the stationary air.
- For the observer on the train, the sound from the front of the train is heard first because he is rushing towards it. The air is rushing backwards, carrying the sound pulse along with it. The train observer measures the sound wave from the front to travel faster than from the back.
- After accounting for this, he agrees with the ground observer that the strikes were simultaneous.
The situation with relativity

- But in relativity, there is no such absolute reference frame.
- There is no such thing as stationary ether.
  The observer on the train sees that the train is stationary, and the ground observer is rushing backwards.
- The observer on the train sees the light pulses from the front and rear travel at exactly the same speed.
- Since they arrive at different times, and he is equidistant between them, he is forced to conclude that they occurred at different times.

Time dilation

- We can even determine exactly how different the time intervals are for different observers.
- This is known as time dilation.
- The interval between two events will be different for different observers, but that difference can be determined.

Time dilation experiment

- Observer O on ground
- Observer O' on train moving at v relative to O
- Pulse of light emitted from laser, reflected from mirror, arrives back at laser after some time interval.
- This time interval is different for the two observers

Why is this?

- Observer O' on train: light pulse travels distance 2d.
- Observer O on ground: light pulse travels farther
- Relativity: light travels at velocity c in both frames
  - Therefore time interval between the two events (pulse emission from laser & pulse return) is longer for stationary observer
- This is time dilation

How large an effect is time dilation?

- \( \Delta t = \) time interval between events in frame O (observer on ground)
- \( \Delta t \) satisfies \( \left( \frac{c \Delta t}{2} \right)^2 = \left( \frac{\sqrt{v^2 - c^2}}{2} \right)^2 + d^2 \).
- \( (\Delta t)^2 \left( c^2 - v^2 \right)^2 = (2d)^2 \)
- \( \Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{1}{c \sqrt{1 - (v/c)^2}} \)

Time dilation

- Time interval in boxcar frame O' \( \Delta t' = \frac{\text{round trip distance}}{\text{velocity}} = \frac{2d}{c} \)
- Time interval in ground frame O
  \( \Delta t = \frac{2d}{c \sqrt{1 - (v/c)^2}} = \frac{\Delta t'}{\gamma} = \gamma \Delta t' \)
  \( \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \)
**Example**

- Suppose observer on train (at rest with respect to laser and mirror) measures round trip time to be one second.
- Observer $O$ on ground is moving at $0.5c$ with respect to laser/mirror.
  $$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.5c/c)^2}} = \frac{1}{\sqrt{1-0.25}} = 1.15$$
- Observer $O$ measures 1.15 seconds

**Which way does time dilation go?**

- The shortest time measured between events is in the frame in which the events occur at the same spatial location.
- This is called the 'proper time' between events, $\Delta t_p$.

Example: The two events could be
1) Minute hand on clock points at '3'
2) Minute hand on clock points at '4'

In the rest frame of the clock, these occur at the same spatial location, and the time interval is 5 minutes.

In frame moving with respect to clock, time interval is $\frac{5}{\gamma} \text{ min}$.
To this observer, the clock is moving and is measured to run slow by factor $\gamma$.

**Time dilation**

- In a time dilation problem, we usually calculate the time interval between two events observed in different reference frames.
- If the events are at the same spatial location in one of the frames...
  - The time interval measured in this frame is called the 'proper time'.
  - The time interval measured in a frame moving with respect to this one will be longer by a factor of $\gamma$.

**The time interval in earth observer’s frame**

- Event #1: leave earth
  $$d = 4.3 \text{ light-years}$$
  $$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.95c/c)^2}} = \frac{1}{\sqrt{1-0.9}} = 0.95$$

- Event #2: arrive star
  $$\Delta t_{\text{earth}} = \frac{d}{v} = \frac{4.3 \text{ light-years}}{0.95c} = 4.5 \text{ years}$$

**The time interval in ship observer’s frame**

- The ship observer measures 'proper time'.
  - Departure, arrival events occur at the same spatial location (in the captain’s chair).
- The proper time $\Delta t_p$.
  - A time interval in a different frame (the earth frame) is related to this one by $\Delta t_{\text{earth}} = \gamma \Delta t_p$

  $$\Delta t_{\text{earth}} = \gamma \Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}}$$

  $$\Delta t_p = \Delta t_{\text{earth}} \sqrt{1-(v/c)^2} = 4.5 \sqrt{1-0.9^2}$$

  $$\Delta t_p = 1.4 \text{ years}$$
But is there a contradiction here?

- Both observer’s agree on the speed (0.95c)
  - Earth observer: ship moving
  - Ship observer: earth and star moving
  - But both agree on the speed
- But if the time intervals are different, and speed is the same, how can distances be the same?
- The distances are not the same! Length contraction.

Length Contraction

- People on ship and on earth agree on relative velocity v = 0.95 c.
- But they disagree on the time (4.5 vs 1.4 years).
- What about the distance between the planets?

Earth frame $d_{\text{Earth}} = v \cdot t_{\text{Earth}} = 0.95 \times (3 \times 10^8 \text{ m/s}) \times (4.5 \text{ years}) = 4 \times 10^{11} \text{ m (4.3 light years)}$

Ship frame $d_{\text{Ship}} = v \cdot t_{\text{Ship}} = 0.95 \times (3 \times 10^8 \text{ m/s}) \times (1.4 \text{ years}) = 1.25 \times 10^{11} \text{ m (1.3 light years)}$

Length contraction and proper length

- Which one is correct?
  - Just like time intervals, distances are different in different frames.
  - There is no preferred frame, so one is no more correct than the other.
- The ‘proper length’ $L_p$ is the length measured in a frame at rest with respect to objects
  - Here the objects are Earth and star.

\[
L = \frac{L_p}{\gamma} = L_p \sqrt{1 - \frac{v^2}{c^2}}
\]