**From Last Time...**
- Events observed to be simultaneous in one frame may not be simultaneous in another.
- Measured interval between events different for different observers:
  - Time dilation. Proper time is that measured in frame where events occur at same spatial location.
  - All other measured times are longer by factor $\gamma$.
- Measured distance between events different for different observers:
  - Length contraction. Proper length is that measured in frame where events are simultaneous.
  - All other lengths are shorter by factor $\gamma$.

**Hour Exam 2**
- Wednesday, Oct. 27
- In-class (1300 Sterling Hall)
- Twenty multiple-choice questions
- Will cover 8.1-8.6 (Light and E&M)
  - 9.1-9.5 (E&M waves and color)
  - 10, 11 (Relativity)
- You should bring
  - 1 page notes, written double sided
  - Calculator
  - Pencil for marking answer sheet

**Time dilation**
I am on jet traveling at 500 mph and throw a ball up and catch it in my hand.
You are on the ground and watch me.
How do the time intervals compare for you and I?

A. $t_{\text{jet}} = t_{\text{Earth}}$
B. $t_{\text{jet}} > t_{\text{Earth}}$
C. $t_{\text{jet}} < t_{\text{Earth}}$

Proper time is measured in the jet frame (events occur at same spatial location).
Times measured in other frames are longer (time dilation).

**Geometrical space-time diagrams**
- We came to these conclusions via particular thought experiments.
- Were able to make quantitative determinations, using sequential logical arguments.
  (e.g. to get from time dilation to length contraction)
- Shortly after relativity, Minkowski developed the space-time diagram.
- A geometrical way to display observations in different reference frames.

**Events and relativity**
- In relativity, it is helpful to think of 'events'
  - E.g. a lightning strike,
  - a flashbulb going off,
  - a spaceship arriving at a planet
- Describe event by spatial position and time as $(x, t)$
- Can think of this as a point in two-dimensional space, but axes would have different units (meters, sec)
- Use event 'coordinates' of $(x, ct)$.
  - Units of $ct = (\text{m}/\text{s})(\text{s}) = \text{meters}$.
- Can now represent events graphically

**The space-time continuum**
- An event is indicated by a point in this graph.
- The time-dependent motion of a particle would be a string of these points.
  - This is called the particle's 'world-line'
### Constant velocity motion
- Worldline of an object moving at constant velocity is a line.

### Scale of the space-time graph
- The axes $ct$ and $x$ both have the same units (meters).
- We scale them so that the world line of a light beam is at a 45° angle.
- For instance, $x$ tick-marks 1 meter apart, $ct$ tick-marks $3 \times 10^8$ meter apart.

### Observing from a new frame
- The point of relativity is that these events will look different in reference frame moving at some velocity.
- Can do this by measuring same event along different coordinate axes.
- Coordinates determined by projecting parallel to $x$ and $ct$ axes (parallel to $x'$ and $ct'$ in new coord system).

### Different inertial frames
- Relativistic effects are apparent when we compare inertial frames moving at different velocities.
- Speed of light constant in all reference frames says tick-marks scale same way on space and time axes.
- Coordinate axes tilt in toward light worldline as relative velocity increases.
- Tick-marks get farther apart as relative velocity increases.

### The train again
- Lightning strikes on train
- Scorch marks on tracks
- “Train” observer
- “Track” observer

### Frame of observer on track
- World lines of objects stationary on the track (moving at velocity $v$ in track frame)
Frame of Observer on Train
Train observer has new coordinate axes.
• $A'$, $B'$, & observer remain at fixed $x'$
• But world lines same in both frames.
• On these new coordinate axes, the original lightning strikes at $A'$ and $B'$ are NOT simultaneous.

Events
• The view here is that the events themselves transcend the observer in the sense that they are fixed on the space-time diagram.
• Different observers record the time and space coordinates differently, but the events themselves have not changed.

Is any measurement the same for all observers?
• Relativity seems to say that there are no more absolutes.
  - Distance between objects depends on observer.
  - Time between events depends on observer.
• But this occurs because we weren't considering the situation as a whole.
• Immutable character of events suggest that there might be invariant quantities.
• Analogy: 2D view of 3D object.

The big picture
• Views of the same cube from two different angles.
• Distance between corners (length of red line drawn on the flat page) seems to be different depending on how we look at it.
• But clearly this is just because we are not considering the full three-dimensional distance between the points.
• The 3D distance does not change with viewpoint.

The real 'distance' between events
• Can do something similar for the space-time continuum
• Need a quantity that is the same for all observers
  - Hint: Speed of light is constant in all frames.
  - World line for light propagation is $x = ct$ (or $x = -ct$)
• A quantity all observers agree on is $x^2 - c^2 t^2 = \text{separation}^2 - c^2 \text{interval}^2$
• Can think of this as the full 'distance' between events.
• Looking at position $x$ or time $t$ separately just confuses things.

The 4D space-time continuum
• Should be considering time as another 'coordinate' which contributes to the 4D 'distance' along with 3 dimensional spatial distance
• Relativisitic invariant for four dimensions is $x^2 + y^2 + z^2 - c^2 t^2$
Example: spaceship prob.

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.95)^2}} = 3.203 \]

Events in the Earth Frame
- Event #1: leave Earth
- Event #2: arrive at star

\[ d = 4.3 \text{ light-years (LY)} \]

\[ \Delta t_{\text{Earth}} = \frac{d}{v} = \frac{4.3 \text{ light-years}}{0.95c} = 4.5 \text{ years} \]

A relativistic invariant quantity

<table>
<thead>
<tr>
<th>Earth Frame</th>
<th>Ship Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event separation = 4.3 LY</td>
<td>Event separation = 0 LY</td>
</tr>
<tr>
<td>Time interval = 4.526 yrs</td>
<td>Time interval = 1.413 yrs</td>
</tr>
<tr>
<td>((\text{separation})^2 - c^2(\text{time interval})^2)</td>
<td>((\text{separation})^2 - c^2(\text{time interval})^2)</td>
</tr>
<tr>
<td>(-4.3^2 - (4.526 \text{ yrs})^2)</td>
<td>(-0 - (1.413 \text{ yrs})^2)</td>
</tr>
<tr>
<td>(-20.0 \text{ LY}^2)</td>
<td>(-2.0 \text{ LY}^2)</td>
</tr>
</tbody>
</table>

- The quantity \((\text{separation})^2 - c^2(\text{time interval})^2\) is the same for all observers.
- It mixes the space and time coordinates.

Time dilation, length contraction

- \(t = \gamma t_{\text{proper}}\)
- \(L = L_{\text{proper}} / \gamma\)

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]

- \(\gamma\) always bigger than 1
- \(\gamma\) increases as \(v\) increases
- \(\gamma\) would be infinite for \(v = c\)

Suggests some limitation on velocity as we approach speed of light.

Addition of Velocities (Non-relativistic)

- Could try to reach higher velocity by throwing object from moving platform.
- Works well for non-relativistic objects.
Relative Velocity (Light)

- Someone ‘throws’ a photon (velocity $3 \times 10^8$ m/s).
  How fast do I think it goes when I am:
  - Standing still $3 \times 10^8$ m/s
  - Running $2 \times 10^4$ m/s towards $3 \times 10^8$ m/s
  - Running $2 \times 10^4$ m/s away $3 \times 10^8$ m/s

Strange but True!

Addition of Velocities (Relativistic)

Relativistic Addition of Velocities

What about intermediate velocities?

Very low velocity: Nonrelativistic

Very high velocity: Extreme relativistic

Relativistic Addition of Velocities

- Galilean relative velocities cannot be applied to objects moving near the speed of light
- Einstein’s modification is
  \[
  v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad}v_{db}}{c^2}}
  \]
  - The denominator is a correction based on length contraction and time dilation

- As motorcycle velocity approaches $c$, $v_{ab}$ also gets closer and closer to $c$
- End result: nothing exceeds the speed of light.