From Last Time...
- Particle can exist in different quantum states, having
  - Different energy
  - Different momentum
  - Different wavelength
- The quantum wavefunction describes wave nature of particle.
- Square of the wavefunction gives probability of finding particle.
- Zero’s in probability arise from interference of the particle wave with itself.

Back to the particle in a box
- Here is the probability of finding the particle along the length of the box.
- Can we answer the question: Where is the particle?

Where is the particle?
- Can say that the particle is inside the box, (since the probability is zero outside the box), but that’s about it.
- The wavefunction extends throughout the box, so particle can be found anywhere inside.
- Can’t say exactly where the particle is, but I can tell you how likely you are to find at a particular location.

How fast is it moving?
- Box is stationary, so average speed is zero.
- But remember the classical version
  - Particle bounces back and forth.
    - On average, velocity is zero.
    - But particle is still moving.
    - Velocity (momentum) even changes sign (direction)

Quantum momentum
- Quantum version is similar.
  - Ground state is a standing wave, made equally of
    - Wave traveling right ( positive momentum $+h/\lambda$ )
    - Wave traveling left ( negative momentum $-h/\lambda$ )
- Can’t say exactly what the momentum is either!

Uncertainty in Quantum Mechanics
Position uncertainty $= L$ 
Momentum uncertainty from $\frac{h}{\lambda} \text{ to } \frac{h}{\lambda} = \frac{2h}{\lambda} \frac{h}{L}$
Reducing the box size reduces position uncertainty, but the momentum uncertainty goes up!
The product is constant: 
(position uncertainty) x (momentum uncertainty) $\sim h$
Heisenberg Uncertainty Principle
• Using
  - $\Delta x =$ position uncertainty
  - $\Delta p =$ momentum uncertainty
• Heisenberg showed that the product
  \[(\Delta x) \cdot (\Delta p)\] is always greater than \(\left(\frac{\hbar}{4\pi}\right)\)

The exact value of the product depends on the problem (particle in a box, pendulum, hydrogen atom)
This is a minimum uncertainty relation.

Understanding the Uncertainty Principle
Sounds mysterious, but this arises because the particle does not have one particular momentum.
It is made up two different waves, with two different momenta.
Similar to our beat frequency problem, combined waves of different frequency, wavelength

We say that the 'beat' wave does not have a particular wavelength, but a 'spread' in wavelengths.

Location of a particle.
Uncertainty in position, uncertainty in wavelength.
Particle is superposition of wavelengths with frequencies from 436 Hz to 440 Hz.
This 'momentum' spread localizes the particle.
'Envelope' corresponds to our notion of a particle.

Probabilistic interpretation
• \(|\Psi(x)|^2dx\) = probability to find particle

Move wave properties
Particle contained entirely within closed tube.
Open top: particle can escape if we shake hard enough.
But at low energies, particle stays entirely within box.
Quantum mechanics says something different!

In quantum mechanics, there is some probability of the particle penetrating through the walls of the box.

Nonzero probability of being outside the box!

The 'Forbidden Region'

• Can also see this in the pendulum.
• Classically, pendulum with particular energy never swings beyond maximum point.
• Quantum wave function extends into classically forbidden region.

An electron in a metal

• Electron is bound inside a metal, as a particle in a well.
• But not entirely!
  - Spatial extent of electron described by quantum-mechanical wavefunction.
  - There is some probability that electron will be found outside of the metal.

Tunneling between conductors

• With two wells very close together, the electron can tunnel between them.
• The probability of this goes down extremely quickly with well separation.
• Two metals need to be within 1-2 nm of each other without touching!

Scanning Tunneling Microscopy

• Over the last 20 yrs, technology developed to controllably position tip and sample 1-2 nm apart.
• Is a very useful microscope!

Can we see atoms?
STM image analysis

STM image analysis

• The tip is scanned across the sample, recording the z-position at each point.

A closer look at a silicon circuit

A closer look at a silicon circuit

Manipulation of atoms

• Take advantage of tip-atom interactions to physically move atoms around on the surface

• This shows the assembly of a circular 'corral' by moving individual iron atoms on the surface of Copper (111).

• The (111) orientation supports an electron surface state which can be 'trapped' in the corral

D. Eigler (IBM)

Quantum Corral

Quantum Corral

• 48 Iron atoms assembled into a circular ring.
• The ripples inside the ring reflect the electron quantum states of a circular ring (interference effects).

The Stadium Corral

The Stadium Corral

Again iron on copper. This was assembled to investigate quantum chaos.

• The electron wavefunction leaked out beyond the stadium too much to observe expected effects.
Some fun!

Kanji for atom (lit. original child)
Iron on copper (111)

Carbon Monoxide man
Carbon Monoxide on Pt (111)

D. Eigler (IBM)

Other materials

• STM relies on electrons tunneling from the tip to the sample.
• The measured quantity is the tunneling current that flows from the tip to the sample.
• This requires a conducting sample.

• But there are lots of interesting materials that are not conducting!

Atomic Force Microscopy

The probe needs to be flexible, but not floppy.
Use micromachined Si or SiN to form cantilever.
Sharpened tip on end

Micromachined Si cantilever for atomic force microscopy

How the AFM works

• The cantilever can ride along, touching the surface atom by atom.
• The deflection is measured by the position of the reflected laser beam.
• Feedback used to adjust z-position to give constant deflection

Organic and biological materials

Contact mode image of human red blood cells.
15µm scan courtesy M. Miles and J. Ashmore, University of Bristol, U.K.
Blood courtesy of Jonathan Ashmore, Professor of Physiology, University College, London.
**DNA and Chromosomes**

- DNA
- Chromosome

155 nm scan. Image courtesy of W. Blaine Stone

**In-situ imaging with AFM**

- Contact AFM (C-AFM) image taken shows the surface of a Rat Basophilic Leukemia -2H3 (RBL-2H3) cell.
- Electron microscopy would have required metal coating and imaging of the sample in vacuum.
- Here the cells were grown on glass cover slips and imaged in-situ while still living.

**Hippopotamus neuron**

- Hippocampus neurons grown 21 days in DEM media on coverslips coated by laminin at 37ºC in CO2/air (3%/95%).
- Sample was fixed in PBS culture with glutaraldehyde and washed with PBS.

**Other types of force microscopy**

- Atomic force microscopy exploited a particular interaction between tip and sample.
- The same force that produces chemical bonds.
- A variety of other forces can be exploited
  - A conducting cantilever can measure the local the local charge density.
  - A magnetic cantilever tip can measure the local magnetic 'topography' of a sample.

**Electric force microscopy**

- Scan metal-coated tip at fixed height above sample
- Use deflection of the cantilever due to electrical interactions to obtain information about the electrical properties

**Positioning charge with the tip**

- Charge from the tip can be locally positioned on the sample.
**Magnetic force microscopy**
- Atomic force microscopy exploited a particular interaction between tip and sample.
- The same force that produces chemical bonds.
- A variety of other forces can be exploited
  - for instance a magnetic force
  - A magnetic cantilever tip can measure the local magnetic "topography" of a sample.

**Magnetic Bits in storage media**
- Magnetic bits written with an MFM probe on perpendicular Co-Cr media with a NiFe sublayer.
- The bits are about 200nm in size spaced 370nm, giving an equivalent area density of 5 Gbits/in².

**Magnetic domains in a thin film**
- MFM image overlayed on tapping-mode topography
- 2 µm x 2 µm scan
- Room temperature
- Red: out of page
- Blue: into page

La₀.7Sr₀.3MnO₃ film grown by off-axis sputtering
Strained for perpendicular magnetization