From Last Time...

- ‘Philosophical’ effects in quantum mechanics
- Interpretation of the wave function:
  - Calculation using the basic premises of quantum mechanics give highly accurate results...
  - ...but what does it mean?
- Superposition of quantum states
- Entanglement, action at a distance
- Copenhagen interpretation
  - ‘collapse of the wavefunction’

Atoms, Molecules and Solids

- Some very basic considerations in quantum mechanics explain some basic properties of atoms, molecules, and solids.
- Electron spin: a new quantum effect
- Indistinguishability: electrons are identical
- Symmetry: qualitative considerations

Additional electron properties

- Free electron, by itself in space, not only has a charge, but also acts like a bar magnet with an N and S pole.
- Since electron has charge, could explain this if the electron is spinning.
- Then resulting current loops would produce magnetic field just like a bar magnet.
- But...
  - Electron in NOT spinning.
  - As far as we know, electron is a point particle.

Electron magnetic moment

- Why does it have a magnetic moment?
- It is a property of the electron in the same way that charge is a property.
- But there are some differences
  - Magnetic moment has a size and a direction
  - It’s size is intrinsic to the electron, but the direction is variable.
- The ‘bar magnet’ can point in different directions.

Quantization of the direction

- But like everything in quantum mechanics, this direction is quantized.
- And also like other things in quantum mechanics, if magnetic moment is very large, the direction quantization is not noticeable.
- But for an electron, the moment is very small.
  - The quantization is a large effect.
  - In fact, there are only two possible directions.
  - We call these spin up and spin down.

Electron spin orientations

- Spin up
- Spin down
Another quantum number

- There is a quantum number associated with this property of the electron.
- Even though the electron is not spinning, the magnitude of this property is the spin.
- The quantum numbers for the two states are:
  - +1/2 for the up-spin state
  - -1/2 for the down-spin state
- The proton is also a spin 1/2 particle.
- The photon is a spin 1 particle.

Indistinguishability

- Another property of quantum particles
  - All electrons are ABSOLUTELY identical.
- Not particularly intuitive.
- On the macroscopic scale, there is always some aspect that distinguishes two objects.
- Perhaps color, or rough or smooth surface
- Maybe a small scratch somewhere.
- Experimentally, no one has ever found any differences between electrons.

Indistinguishability and QM

- Quantum Mechanics says that electrons are absolutely indistinguishable.
  - Treats this as an experimental fact.
    - For instance, it is impossible to follow an electron throughout its orbit in order to identify it later.
- We can still label the particles, for instance
  - Electron #1, electron #2, electron #3
- But the results will be meaningful only if we preserve indistinguishability.

Consequences

- This has some amazing consequences.
- For electrons, it leads directly to the conclusion that only one electron can be put in each quantum state.
- This is what makes atoms be different.
- It is what gives metals the properties we observe.

How can this be?

- How can we label particles, but still not distinguish them?
- What is really meant is that no physically measurable results can depend on how we label the particles.
- One physically measurable result is the probability of finding an electron in a particular spatial location.

Example: 2 electrons on an atom

- Probability of finding an electron is given by the square of the wavefunction.
- We have two electrons, so the question we would is ask is
  - How likely is it to find one electron at location \( r_1 \) and the other electron at \( r_2 \)?
• Suppose we want to describe the state with one electron in a 3s state and one electron in a 3d state.

![Diagrams of 3s and 3d states]

On the atom, they look like this. (Both on the same atom.)

Probabilities

• The probability of finding the particles at particular locations is the square of the wavefunction.
• Indistinguishability says that these probabilities cannot change if we switch the labels on the particles.
• However the wavefunction could change, since it is not directly measurable. (Probability is the square of the wavefunction)

Two possible wavefunctions

• Two possible symmetries of the wavefunction, that keep the probability unchanged when we exchange particle labels:
  - The wavefunction does not change
    Symmetric
  - The wavefunction changes sign only
    Antisymmetric

In both cases the square is unchanged

The symmetric state

![Symmetric state wavefunction diagram]

The antisymmetric state

![Antisymmetric state wavefunction diagram]

Spin-statistics theorem

• In both cases the probability is preserved, since it is the square of the wavefunction.
• Can be shown that
  - Integer spin particles (e.g. photons) have symmetric wavefunctions
    These types of particles are called Bosons
  - Half-integer spin particles (e.g. electrons) have antisymmetric wavefunctions
    These types of particles are called Fermions
So what?
- Fermions - antisymmetric wavefunction:

![Wavefunction Diagram](image)

Try to put two Fermions in the same quantum state (for instance both in the s-state)

![Fermion States Diagram](image)

Pauli exclusion principle
- Only wave function permitted by symmetry is exactly zero.
- Cannot put two Fermions in same quantum state
- This came entirely from indistinguishability, that electrons are identical.
- Without this, there elements would not have diff. chem. props., properties of metals would be different, neutron stars would collapse.

Include spin
- We labeled the states by their quantum numbers. One quantum number for each spatial dimension.
- Now there is an extra quantum number: spin.
- A quantum state is specified by it’s space part and also it’s spin part.
- An atom with several electrons filling quantum states starting with the lowest energy, filling quantum states until electrons are used.

Putting electrons on atom
- Electrons are Fermions
- Only one electron per quantum state

- Hydrogen: 1 electron
  - one quantum states occupied
  - n=1 states

- Helium: 2 electrons
  - two quantum states occupied
  - n=1 states

Other elements
- More electrons requires next higher energy states
- Lithium: three electrons

- n=2 states, 8 total

- n=1 states
  - one spin up, one spin down

Elements with more electrons have more complex states

Bosons
- No exclusion principle for integer spin particles such as photons (symmetric wave function)

- Any number of photons can be in a single quantum state.
- Above are two photons in the same quantum state.
LASER

- **Light Amplification by Stimulated Emission of Radiation**
- Laser is a device that produces "coherent" light
- Light is emitted when atoms make transition from an excited state to a lower energy state.

- Why does this happen? Why does the atom drop from one state to another

Stimulated emission

- An incoming photon can trigger an atom to make the transition.
- Can think of this as:
  - Electric field from photon induces charge oscillation
  - An oscillating charge will produce radiation
  - This radiation is in phase with the incoming photon.

Laser operation

- Some external power puts energy into atoms to excite them to a higher quantum state.
- Atoms sit, waiting for an incoming photon to trigger the atomic transition.
- Transition produces another photon, exactly in phase, and with same frequency, as incoming photon.
- This process builds up a coherent (in-phase) light beam that eventually exits the laser.

Ruby Laser

- Ruby crystal has the atoms which will emit photons
- Flashtube provides energy to put atoms in excited state.
- Spontaneous emission creates photon of correct frequency, amplified by stimulated emission of excited atoms.

A solid state ‘diode’ laser

- Excitation caused by current passing through semiconductor structure.

Symmetry

- Symmetry is very important in determining qualitative behavior in quantum mechanics.
- Arguments are very general.
- As in the Fermion / Boson argument
  - Wavefunction is not measurable
  - Charge density is measurable (charge dens = prob of finding electron = square of wavefunction)
A two atom molecule

One electron orbiting two atoms

What do we expect for the charge density?
If atoms are identical, do we expect more charge on right, left?
No reason to expect anything other than symmetric charge distribution

Two possible wavefunctions: sym. & antisym.

Solids

- Solids consist of many atoms bonded together in this way.
- Many possible ways to combine atomic wavefunctions to get charge density with correct symmetry.
- All these quantum states have slightly different energy, due to the slightly different charge densities.
- So the solid is similar to the an atom or molecule, except quantum states are extremely close together in energy.