Homework ch 1

1-12. \((x_0', t_1') \Rightarrow t_1 = \gamma (t_1' + \frac{Vx_0'}{c^2})\)

\((x_0', t_1') \Rightarrow t_2 = \gamma (t_2' + \frac{Vx_0'}{c^2})\)

so \(t_2 - t_1 = \gamma (t_2' - t_1')\)

b) don't need to solve for \(x_0\)

1-19. \(\gamma = \frac{1}{\sqrt{1 - (0.9)^2}} = 0.9945\)

\(\text{in } S' \quad u' = \frac{u'' - (-0.9)}{1 - u''(-0.9) - \frac{c}{c}} = 0.9945c\)

\(\text{in } S \quad u = \frac{u' - (-0.9)}{1 - u'(-0.9)} = \frac{1.8945}{1 + (0.9)(0.9945)} = 0.9997c\)

1-26. 27) in earth \(t = \frac{t}{V} = \frac{4.62}{1.75c} = 5.33 \text{ yr}\)

b) \(t' = \frac{t}{V} = 5.33 \text{ yr} / 1.51 = 3.53 \text{ yr}\)

Another way

spacetime coords are \((x,t) = (4.62, 5.33 \text{ yr})\)

so \(t' = \gamma (t - \frac{Vx}{c}) = 1.51 \left( 5.33 - \frac{1.75(4)}{c} \right) = 3.53 \text{ yr}\)
Axis calibration for t':

If l is length scale for t
and l' for t'

\[ t' = \frac{t l}{\cos \theta} = \frac{t l'}{y} \]

so

\[ l = l' \frac{y}{\cos \theta} \]

\[ = l \sqrt{1 + \frac{y^2}{c^2}} \]

\[ = 1.89l \]

\[ (\Delta s)^2 = (5 \mu \text{sec})^2 (2400 \text{m})^2 = -(1873 \text{m})^2 \quad \text{spacelike} \]

moving frame \[ \vec{v}' = \begin{pmatrix} 2400 \mu \text{sec} \end{pmatrix} \quad \gamma(T - \frac{V}{c^2}) \]

so

\[ 1 = \gamma \left( T - \frac{V}{c^2} \right) \]

or

\[ \frac{V}{c} = \frac{Tc^2}{\gamma (1 + \frac{V}{c^2})} = 1.875 \times 10^8 \text{m/s} = 0.625c \]

Solve for \( V \) (quadratic formula) \( \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \)

try some numbers \( V = .1 \quad \frac{V}{c} = .062c \quad \text{got lucky!} \)

\[ V = .899 \Rightarrow .625c \]

\( \therefore V = .999c \) moving to the right. Note that the "distance" between the two emissions is still 2400m, but the light pulses are emitted at different times.
5) \[ \theta \]

Easy way: the light has a velocity vector
\[ \mathbf{u} = c \left( \cos \theta, \sin \theta \right) = c \left( \frac{\sqrt{3}}{2}, 1 \right) \]

while \[ \mathbf{v} = c \left( \frac{\sqrt{3}}{2}, 0 \right) \]

\( \theta \) in moving frame: \[ u_x' = 0, \quad u_y' = c \]

\( \theta \) light moves at \( \theta = 90^\circ \) in moving frame.

General way:
\[ u_x' = \frac{u_x - v}{\gamma \left( 1 - \frac{u_x v}{c^2} \right)} = \frac{c \cos \theta - v}{1 - \frac{v}{c} \cos \theta} \]

\[ u_y' = \frac{u_y}{\gamma \left( 1 - \frac{u_x v}{c^2} \right)} = \frac{c \sin \theta}{\gamma \left( 1 - \frac{v}{c} \cos \theta \right)} \]

\[ \tan \theta' = \frac{u_y'}{u_x'} = \frac{c \sin \theta}{\gamma \left( c \cos \theta - v \right)} \]

\( \theta = 30^\circ \), \[ v = \sqrt{\frac{3}{4}} c \] \( \Rightarrow \tan \theta' = \infty \Rightarrow \theta' = 90^\circ \)

6) \( t = t' = 0 \)

\[ \begin{array}{c}
\text{(a)} \text{ in rocket frame,} & t = \frac{t'}{\frac{v}{c}} = \frac{200\text{ m sec}}{0.68c} = 0.98\mu\text{sec} \\
\text{(b)} \text{ signal received} & t = \gamma \left( t_c' + c \right) = \gamma \left( 1.65\mu\text{sec} + \frac{0.68c}{c} \right) = 2.25\mu\text{sec} \\
\text{(c)} \text{ signal received later than (a), so} & \frac{t}{c} + \frac{t}{c} = 1.65\mu\text{sec} \\
\text{(d)} \text{ signal received} & \gamma \left( x_c + vt_c' \right) = \gamma \left( 0 + 0.68c \times 1.65\mu\text{sec} \right) = 459\text{ m} 
\end{array} \]
7) a) \( \delta \geq V = 1.07 \cdot 26 \text{ns} = 1.36 \text{c} = 3.0 \text{m} \)

b) \( t = \frac{x}{b} = \frac{3.0 \text{m}}{1.36 \text{c}} = 27.8 \text{ns} \)

c) \( \frac{1}{2} \) decays occur at \( x', t' = (0, 26 \text{ns}) \)

\( \text{Note that the distance seen by the atom is shortened } \left( \frac{2.8}{\frac{1}{\gamma}} = \frac{2.8}{1.07} = \frac{1}{1.07} \right). \)

8) a) \( L' = \frac{L}{\gamma} = \frac{1.06}{1.25} = 0.8 \text{m} \)

b) If they are equal lengths, they must move at equal speeds in the new frame. Transforming from a frame where one stick is at rest and the other has velocity \( 0.6 \text{c} \), to a frame moving with velocity \( V \)

\( \left| -V \right| = \frac{U - V}{1 - \frac{UV}{c^2}} \Rightarrow V = \frac{c}{U} \left( c - \sqrt{c^2 - u^2} \right) \)

\[ V = \frac{c^2}{U} \left( 1 - \frac{1}{\gamma} \right) = 0.333 \text{c} \]

\( \frac{L}{\gamma} = \frac{1.06}{1.07} = 0.94 \text{m} \)