PROBLEM SET 5. SOLUTIONS

Source of Galactic Magnetic Field: (20 pts) In class, we have discussed dynamos, but dynamos amplify fields which already exist. Except for the Biermann battery effect treated in Problem Set 1, we have said little about the ultimate origin of magnetic fields. This problem is intended to address that gap. Do not expect to solve it exactly, but state all your assumptions clearly.

1. Refer back to Problem Set 1. Argue that the Biermann battery generates magnetic field more efficiently at small scales than at large scales. Solution: The magnetic field is generated by the battery electric field \( E = -(\nabla P_e)/en_e \). Substituting this into Faraday’s law and using the equation of state we see that the rate of induction is proportional to \( \nabla n_e \times \nabla T_e \). If the gradient lengthscales for the density and temperature are \( L_n \) and \( L_T \), the field will scale as \( (L_n L_T)^{-1} \). Thus, the field grows most quickly at the small scales. This means that small objects, such as stars or accretion disks, are much more quickly magnetized than large objects like galaxies.

2. Based on Part 1, suppose the Galactic magnetic field arose from many small sources, which generated the field and then spewed it into the interstellar medium. For example, the Crab Nebula, a young supernova remnant of the type known as a plerion, is about 2 pc in size and its magnetic field is about \( 10^{-3} \)G. Imagine that the nebula disperses and eventually reaches pressure balance with the interstellar medium. Assuming an interstellar pressure of \( 10^{-12} \) dynes cm\(^{-2} \), what is the size to which the nebula expands, and what is the field strength? Assume all the pressure in the nebula is magnetic. Solution: The magnetic field strength which is in pressure balance with the interstellar gas is about 5.0 \( \mu \)G. Assuming that the nebula expands spherically, \( B \) will scale with radius \( R \) as \( R^{-2} \). Thus, to reduce the field by a factor of 200, \( R \) must expand by a factor of 14.1, to 28.2 pc.

3. It is reasonable, based on what we know about the supernova rate in the early galaxy, to assume that \( 10^6 \) Crab nebulae were present. What is the mean field in the Galactic disk resulting from their superposition? Hint: There is no evidence that magnetic fields in supernova remnants are correlated from one remnant to another. Solution: Using the results of Part 1, the total volume occupied by pressure-equilibrated
Crab nebulae is about $1.2 \times 10^{10} \text{pc}^3$, while the volume of the magnetized Galactic disk is about $3 \times 10^{11} \text{pc}^3$. Thus, we have to assume that the field is dispersed by some sort of diffusive process which mixes it with the gas. The mean separation between the sources is 66 pc. If the field spreads to this size it is reduced to $1.8 \mu G$. However, the fields from separate sources are randomly oriented with respect to one another, so we reduce this by $\sqrt{N} = 10^3$. This gives a “seed” field of $1.8 \times 10^{-9} \text{G}$. This is only an estimate, not a rigorous calculation.

4. Based on the strength and configuration of the field produced by this multiple source mechanism, is this a reasonable model of the Galactic magnetic field, or is a dynamo required? **Solution:** According to this model, the mean field, which is only a statistical fluctuation, is much smaller than the observed mean galactic field, and the random field is much less than the mean. Thus, an additional process, such as a dynamo, must act on the field.


**Summary: (15 pts)** Write a short essay (1 page or less) summarizing the main features of the Galactic magnetic field which a theory of its origin and evolution should explain. These are the most important points to carry away from this part of the course. **Solution:** Important features to mention are:

1. Direction & orientation are primarily azimuthal, in the direction of Galactic rotation with a small modification due to flow along the spiral arms.
2. Large scale (coherent over several kpc) field and small scale (random) fields are comparable in strength.
3. The magnetic energy density in the field is comparable to that in turbulent motions.
4. The large scale field reverses a few times with galactocentric radius, apparently in the regions between spiral arms.
5. There is a strong, well ordered, vertical field near the Galactic Center.
6. Disk galaxies similar to the Milky Way are observed to have similar fields, and fields were present when galaxies were less than a tenth of their present age.