Calculating particle properties of a wave

- A light wave consists of particles (photons):

  The energy *E* of the particle is calculated from the frequency f of the wave via Planck: $\boxed{\mathbf{E = \mathit{h}\, f}}$ (1)

- A particle can act like a wave:

  The momentum p of the particle is calculated from the wavelength λ via de Broglie: $\boxed{\mathbf{p = \mathit{h}/\lambda}}$ (2)

- Frequency f and wavelength λ can be converted into each other by the wave equation: $\boxed{\lambda \cdot \mathbf{f = v}}$ (3)

Energy of a photon

- Consider light with a wavelength $\lambda = 500\ nm$
- Use (1),(3), v=*c*

$$\underset{(1)}{E = hf} \underset{(3)}{=} \frac{hc}{\lambda} = \frac{\left(6.634 \cdot 10^{-34} J \cdot s\right) \cdot \left(3 \cdot 10^{8} m/s\right)}{500 \cdot 10^{-9} m} = 4 \cdot 10^{-19} J \quad = \quad 2.5\ eV$$

A practical energy unit in quantum physics is an *eV*, the energy of an electron accelerated by 1 *Volt* :

$$eV = \textit{electron-Volt} = (\text{electron charge } e\ ) \cdot (1\ \textit{Volt})$$
$$= (1.6 \cdot 10^{-19}\ \textit{Coulomb}) \cdot (1\ \textit{Volt})$$
$$= \ 1.6 \cdot 10^{-19}\ \textit{Joule}$$

Conversion of *Joule* to *eV* : $\boxed{J = 6.24 \cdot 10^{18}\ eV}$

Conversion between wavelength and energy

Wavelength and energy are inversely proportional with the proportionality constant hc:

$$E = hc/\lambda$$

$$\lambda = hc/E \qquad hc = 1240\ eV \cdot nm$$

Green light for example:

$$E = 1240\, eV \cdot nm\ /\ 550\, nm = 2.25\, eV$$

How many photons can you see?

In a test of eye sensitivity, experimenters used 1 millisecond flashes of green light. The lowest light power that could be seen was $4 \cdot 10^{-14}$ *Watt*.

How many green photons (550 *nm*, 2.25 *eV*) is this?

A. 10 photons
[✓ B. 100 photons]
C. 1,000 photons
D. 10,000 photons

$10^{-3}\,s \cdot 4 \cdot 10^{-14}\,W = 4 \cdot 10^{-17}\,J =$

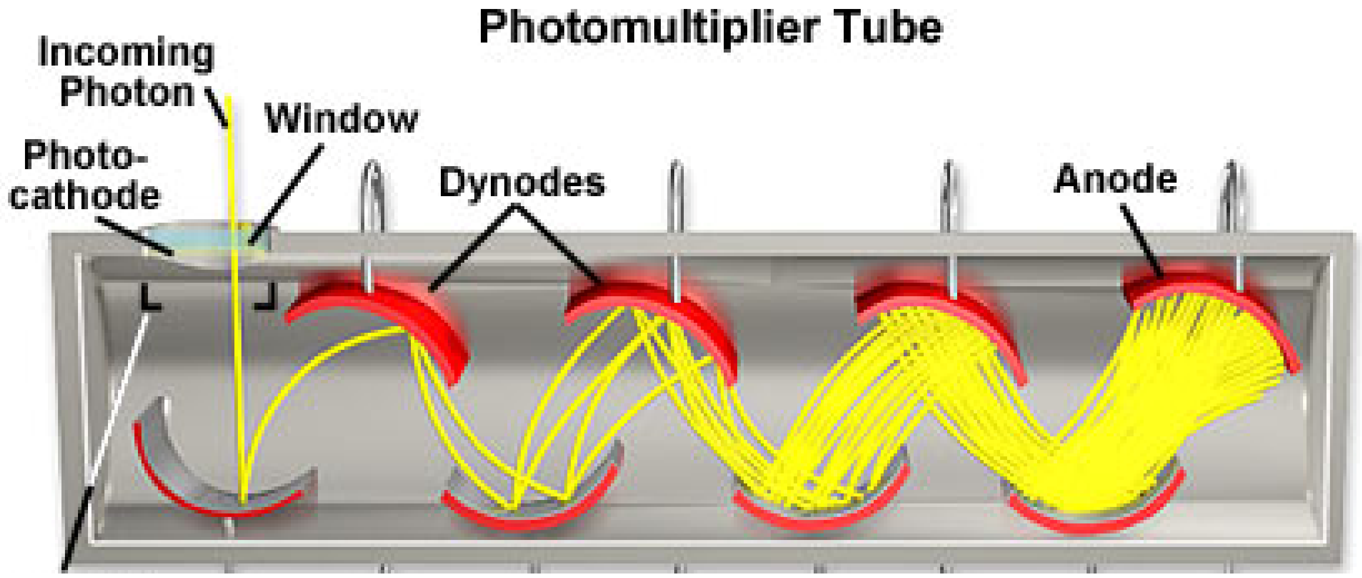
$6.24 \cdot 10^{18} \cdot 4 \cdot 10^{-17}\,eV = 250\,eV;$

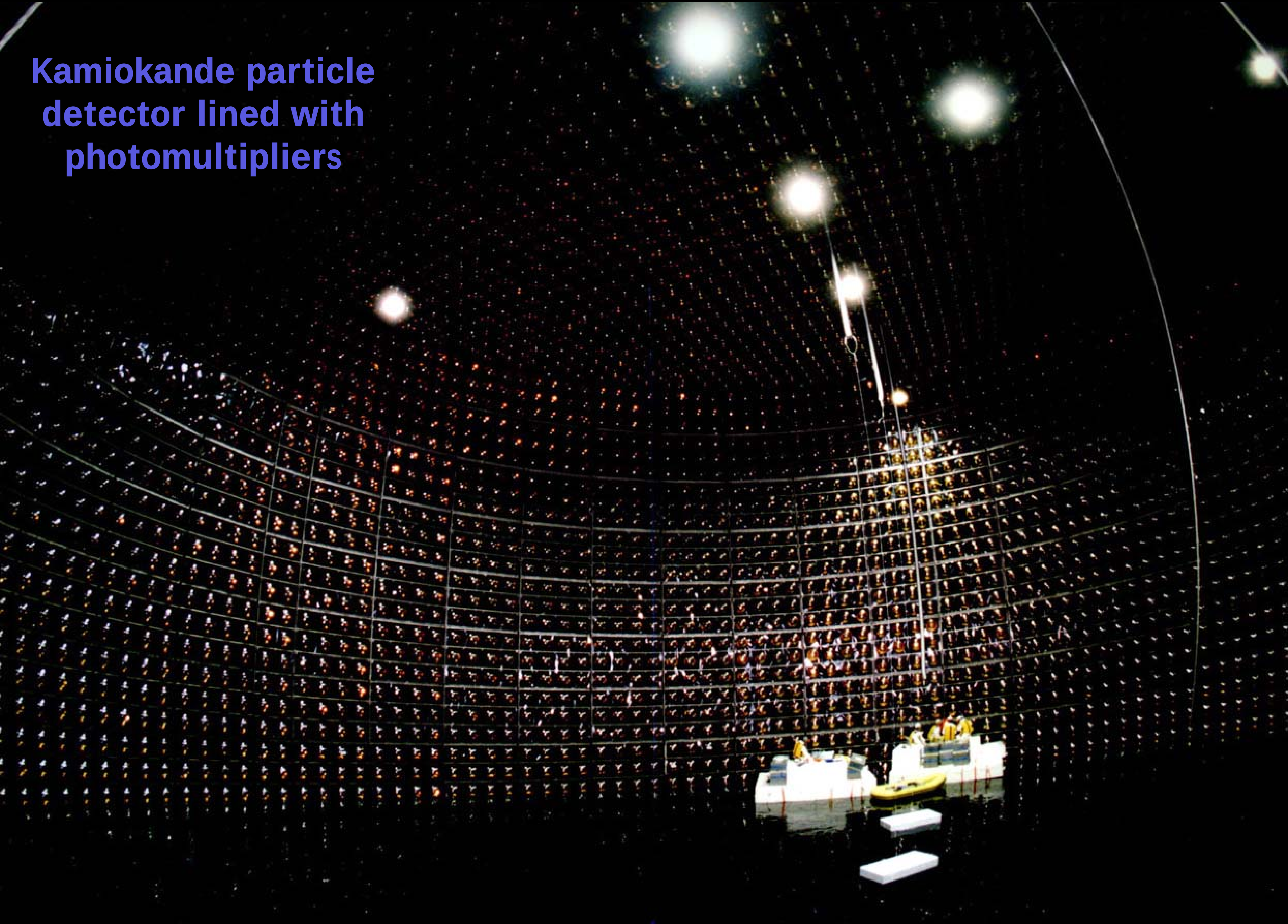
$250eV / 2.25eV = 111$ photons

Detecting single photons

A photon can be converted into a pulse of electrons:

- For ultraviolet light, X-rays, Gamma rays: In a Geiger counter a photon knocks electrons from gas molecules, creating a miniature spark.
- For visible light: In a photomultiplier a photon kicks an electron out of a solid (the photocathode). The electron is multiplied into a million electrons by multiple bounces.





Kamiokande particle detector lined with photomultipliers

The wave character of electrons

$$\lambda = \frac{h}{p} = \frac{h}{m_e \mathrm{v}} = \frac{6.6 \cdot 10^{-34}\, Js}{\left(9 \cdot 10^{-31} kg\right) \cdot \left(velocity\right)}$$

- The wavelength is inversely proportional to the velocity v of the electron and to its mass m_e.
- An electron with low velocity (low kinetic energy) has a long wavelength, which is easier to detect.

Wavelength of an electron from its kinetic energy

1) Velocity v from the kinetic energy E_{kin}:

$$E_{kin} = \tfrac{1}{2}\, m_e v^2 \qquad v = \sqrt{2\, E_{kin}/m_e}$$

2) Wavelength λ via de Broglie and $p = m_e v$:

$$\lambda = h/m_e v$$

For a kinetic energy $E_{kin} = 1\,eV$ one obtains:

1) $v = \sqrt{2 \cdot 1.6 \cdot 10^{-19}\, J \;/\; 9 \cdot 10^{-31}\, kg}$ $\qquad (eV = 1.6 \cdot 10^{-19} J)$

$= 6 \cdot 10^{5}\, m/s$ $\qquad (m_e = 9 \cdot 10^{-31}\, kg)$

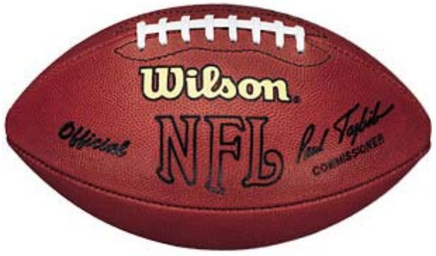
2) $\lambda = 6.6 \cdot 10^{-34} Js \,/\, 9 \cdot 10^{-31}\, kg\; 6 \cdot 10^{5}\, m/s$ $\qquad (h = 6.6 \cdot 10^{-34} Js)$

$= 1 \cdot 10^{-9}\, m \;=\; 1\, nm$

Typical wavelength of an electron

- The wavelength of an electron is very short and hard to measure with macroscopic tools.
- The spacing of nickel atoms in a crystal was needed to demonstrate electron diffraction.
- A typical electron energy in a solid with 1eV energy has a wavelength of only 1 nanometer.
- Nanotechnology has the capability to shape electron waves and thereby influence the behavior of electrons in a solid (Lect. 24a).

Wavelength of a football



NFL guidelines: “Weight 14 to 15 ounces” = *0.4 kg*
Brett Favre can throw the ball 70 miles/hour = 30 m/s

$$mv = (0.4\ \ kg)(30\ \ m/s) = 12\ \ kg \cdot m/s$$

$$\lambda = \frac{h}{p} = \frac{6.6\ \cdot 10^{-34}\ \ J \cdot s}{12\ \ kg \cdot m/s} = 5.5\ \cdot 10^{-35}\, m$$

That is similar to the Planck length where space-time falls apart, 10^{14} times smaller than the smallest measurable length (Lect. 3).

Conclusion: **Macroscopic objects don’t show measurable quantum effects.**

When are quantum phenomena important ? (optional)

1) Quantum physics is important at small distances d, smaller than the de Broglie wavelength $\lambda = h/p$: $\boxed{d < \lambda}$

Example: Nanotechnology for shaping electron waves.

2) Quantum physics is important for large energy quanta $E = h f$:

Example: Planck's radiation law cuts the spectrum off when the energy to create a photon exceeds the available thermal energy ($E_{\text{therm}} \approx 0.1\ eV$ at T=300K) : $\boxed{E > E_{\text{therm}}}$

Example: The photoelectric effect occurs only when the photon energy exceeds the minimum energy Φ to knock out an electron (Φ = "work function" $\approx$ 4eV) : $\boxed{E > \Phi}$

Criteria 1) and 2) are connected by the wave equation:

$\lambda \cdot f = v$ (v = velocity of the wave)

If the wavelength λ is small the frequency f is large and the energy quantum $E = hf$ is large .

Example:

Quantum computing requires that quantum effects are dominant.

Three approaches to quantum computing are pursued in the Physics Department (silicon quantum dots, superconducting junctions, and cold atoms in traps). All of them use very low temperatures to keep the quantum effects undisturbed by thermal noise (Criterion 2).