The A_y Problem for p-^{3}He Elastic Scattering

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We present evidence that numerically accurate quantum calculations employing modern internucleon forces do not reproduce the proton analyzing power, A_y, for p-^{3}He elastic scattering at low energies. These calculations underpredict new measured analyzing powers by approximately 30% at $E_{\text{c.m.}} = 1.20$ MeV and by 40% at $E_{\text{c.m.}} = 1.69$ MeV, an effect analogous to a well-known problem in p-d and n-d scattering. The calculations are performed using the complex Kohn variational principle and the (correlated) hyperspherical harmonics technique with full treatment of the Coulomb force. The inclusion of the three-nucleon interaction does not improve the agreement with the experimental data.

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Over the past decade, numerical calculations for three-nucleon systems have reached a high degree of precision. This has made it possible to carry out accurate quantum mechanical computations for a variety of processes including (i) N-d elastic scattering and breakup [1,2], (ii) N-d radiative capture [3,4], (iii) photodisintegration of ^{3}H and ^{3}He [5,6], and (iv) electron-^{3}H and electron-^{3}He scattering [4,7] (see Ref. [8] for a more complete list of references). The calculations use a variety of approaches [8], such as the Kohn variational principle, the Green Function Monte Carlo method, or direct solution of the Faddeev equations, and have made it possible to test our knowledge of the pairwise nucleon-nucleon (NN) interaction and to study effects of possible three-nucleon (3N) forces.

Calculations employing modern NN and 3N interactions are capable of describing most of the experimental results for the processes listed above. However, there is a well-known and large discrepancy for the N-d analyzing power at low energies. What one finds is that the predicted A_y values are significantly smaller in magnitude than the measurements for both p-d and n-d elastic scattering. Resolving this "A_y puzzle" is a current and important area of research [1,9].

In the present Letter, we extend some of the analyses described above to the four-nucleon system. We will present new accurate computations for p-^{3}He elastic scattering at low energies. We will also report new measurements of the analyzing power for p-^{3}He scattering which have been obtained for the purpose of testing whether the A_y problem occurs in this system as well.

Extending the accurate quantum calculations into the 4N system is obviously of importance since it allows many new and stringent tests of the nuclear interaction. For example, accurate calculations of the alpha particle binding energy B_4 have been achieved recently [10–12], and it has been found that the experimental value of the binding energy is reproduced with the same NN and 3N potentials that fit the ^{3}H binding energy. Since it appears that no four-nucleon potential is necessary to explain the α-particle binding energy, one might expect that NN + 3N interactions alone would also be sufficient to describe the various four nucleon scattering processes.

The development of techniques for solving 4N problems is also important for other reasons. Many reactions involving four nucleons, such as $d + d \rightarrow {}^4\text{He} + \gamma$ or $p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e$ (the *hep* process), are of extreme interest from the astrophysical point of view. The theoretical description of these processes constitutes a challenging problem from the standpoint of nuclear few-body theory. Its difficulty can be appreciated by considering, for example, the *hep* process. In Ref. [13] it was found that the capture from the initial P-wave channels ("forbidden" transitions) gives about 40% of the calculated S factor, and this fraction depends critically on the correct description of the dynamics of the continuum and bound 4N states.

Moreover, the study of 4N systems is important also for testing the various many-body techniques developed for studying systems with large ($\geq$4) numbers of particles. These theories necessarily include a number of approximations whose consequences can be investigated by comparison with more sophisticated calculations. The 4N system is the simplest system in which these checks can be advantageously performed.

The new calculations reported in this Letter are based on the Kohn variational principle and make use of correlated hyperspherical harmonic (CHH) functions. The same approach has already been applied successfully for a variety of 3N processes [2,14]. A previous application for studying 4N scattering at zero energy was already reported [15]. In the present paper, we have improved our calculation to determine P- and D-wave phase shifts. The convergence of the P-wave shifts is rather slow and has required a large technical effort to be achieved. Results will be reported for calculations based on (i) the Argonne V18 NN potential

[16] (the AV18 model), and (ii) the Argonne V18 NN potential plus the Urbana IX 3N potential [10] (the AV18UR model). The present calculations represent the first attempt to study the effects of 3N forces on the p-^{3}He scattering observables at energies greater than zero. In this paper we focus on low energies where the convergence properties of our theoretical approach are more satisfactory and where meaningful comparisons with the experimental data can be performed.

The new measurements of the proton analyzing power A_y for p-^{3}He elastic scattering were obtained at $E_{\mathrm{c.m.}} = 1.20$ and 1.69 MeV. The experiments were carried out at the University of Wisconsin tandem accelerator laboratory. Polarized protons from a crossed-beam polarized ion source [17] were accelerated, momentum analyzed by a 90° bending magnet, and transported to a 1-m scattering chamber. The scattering chamber was filled with 43.4 Torr of 99.95% purity ^{3}He gas, and was isolated from the beam line vacuum by a 4.44×10^{-5} cm thick Ni entrance foil located 1.27 cm from the chamber center.

Elastically scattered protons were detected by three rectangular silicon surface-barrier detectors, 60 to 100 μm thick, placed symmetrically on each side of the scattering chamber. A slit assembly restricted the angular field of view to $\pm 0.34°$. The spectra were clean except for a small contaminant peak that was well separated from the peak of interest except at the most forward angle. At that angle, background subtraction was performed.

After passing through the scattering chamber, the beam entered a polarimeter in which the beam polarization was determined using $\vec{p}$-α elastic scattering [18]. The polarimeter was filled with one-half atmosphere natural He gas, and was separated from the scattering chamber by a 2.54×10^{-4} cm thick Havar foil. Because of the low beam energies, we could not measure the beam polarization very accurately at the same time as data were being taken. However, previous experience indicates that the beam polarization does not normally change significantly over time. Consequently, at least once a day we made a careful measurement of the beam polarization by increasing the beam energy to 4.0 MeV at the center of the polarimeter. At this energy, the polarimeter analyzing powers are known to 2%. Each such careful measurement of the beam polarization yielded a value between 0.79 and 0.84 with typical statistical errors of ± 0.01.

The new measurements are shown in Fig. 1. The error bars include statistical uncertainties and also at the extreme forward angle an estimate of uncertainty in the background subtraction. There is also a scale factor uncertainty of 3% due to beam polarization uncertainties.

We turn now to the theoretical calculations. Four-nucleon scattering problems have been studied theoretically for a long time (see Ref. [19], and references cited therein). Recently, increases in computing power have opened the possibility for accurate calculations of the 4N observables using realistic models for NN forces. These

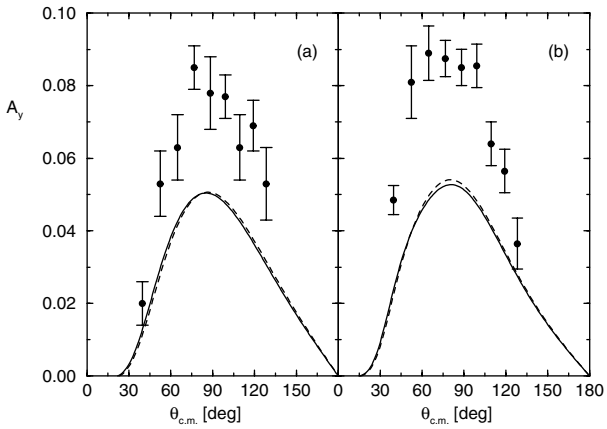

FIG. 1. Measurements of the proton analyzing power A_y as a function of the scattering angle at $E_{\mathrm{c.m.}} = 1.20$ MeV (panel *a*) and 1.69 MeV (panel *b*). The theoretical estimates obtained with the AV18 (solid curves) and the AV18UR (dashed curves) interaction models are also reported.

calculations have been performed mainly by means of the Faddeev-Yakubovsky (FY) approach [20,21] and the Kohn variational principle [15,22]. In this Letter, the wave functions of the scattering states are expanded in terms of the CHH basis [14] and the complex form of the Kohn variational principle is applied [23,24].

The wave function (WF) of a 4N state with total angular momentum J, parity Π, and total isospin T, T_z (in the present case $T = T_z = 1$) can be written as [15]

$$\Psi^J_{LS} = \Psi^J_C + \Phi^J_{LS}, \qquad \Pi \equiv (-)^L. \tag{1}$$

The first term Ψ^J_C of Eq. (1) must be sufficiently flexible to guarantee a detailed description in the "internal region," where all the particles are close to each other and the mutual interaction is large; Ψ^J_C goes to zero when the distance r_i between the ^{3}He and the unbound proton i increases. This term in the WF is expanded in terms of CHH basis functions, following the procedure discussed in detail in Ref. [25].

The second term Φ^J_{LS} describes the asymptotic configuration of the system, for large r_i values, where the nuclear p-^{3}He interaction is negligible. Let us introduce the surface functions

$$\Omega^{(\lambda)}_{LSJ} = \sum_{i=1}^{4} \{Y_L(\hat{r}_i)[\Psi^{^3\mathrm{He}}_{jk\ell}\chi_i]_S\}_{JJ_z} \mathcal{R}^{(\lambda)}_L(r_i), \tag{2}$$

where χ_i is the spin function of the unbound nucleon i and $\Psi^{^3\mathrm{He}}_{jk\ell}$ is the ^{3}He bound state WF. This latter function is normalized to unity and is antisymmetric under the exchange of any pair of particles j, k, and ℓ. $\Psi^{^3\mathrm{He}}$ has been determined as discussed in Ref. [14] by using the CHH expansion for a three-body system.

The functions $\mathcal{R}^{(\lambda)}_L(r_i)$ of Eq. (2) are the ingoing ($\lambda \equiv -$) and outgoing ($\lambda \equiv +$) radial solutions of the

two-body Schrödinger equation without nuclear interaction [24]. The asymptotic WF is then written as

$$\Phi^J_{LS} = \Omega^{(-)}_{LSJ} - \sum_{L'S'} {}^J S^{SS'}_{LL'} \Omega^{(+)}_{L'S'J}, \qquad (3)$$

where the S-matrix elements ${}^J S^{SS'}_{LL'}$ give the amplitude of the outgoing $(L'S')$ component relative to the ingoing (LS) wave. The elastic S matrix, whose dimensionality is 1 (2) for the $J = 0$ ($J > 0$) states, should be unitary since there are no open reaction channels at the energies considered here. It follows that the eigenvalues of the S matrix are written as $\exp(2i\delta^J_{LS})$, where δ^J_{LS} is the eigenphase shift of the ${}^{2S+1}L_J$ wave. These quantities are calculated by means of the complex form of the Kohn variational principle with a procedure similar to that one used in the N-d case [14,24].

Note that with the present method the unitarity of the S matrix is assured only after the complete convergence of the CHH expansion. Thus, for example, in the case of the $J = 0$ waves, the calculated S matrix will be of the form ${}^0 S^{SS}_{LL} = \eta^{J\Pi} \exp(2i\delta^0_{LS})$ where the "inelasticity" parameter $\eta^{J\Pi}$ may differ from 1 if the convergence is incomplete. For $J > 0$ states, the inelasticity parameter can be defined as $\eta^{J\Pi} = \sqrt{\mathrm{Tr}[S^\dagger \cdot S]/2}$.

The expansion of the internal part Ψ^J_C is conveniently studied by grouping the functions of the basis in "channels" (a given channel contains CHH states with the same angular-spin-isospin quantum numbers). The convergence of the $L = 0$ waves ($J^\Pi = 0^+, 1^+$) at $E_{\mathrm{c.m.}} = 0$ was studied previously in Ref. [15]. At $E_{\mathrm{c.m.}} = 0$ and at the energies considered here, a rather small number of channels is sufficient to provide a good convergence. This is due mainly to the Pauli principle which limits overlaps between the four nucleons. As a consequence, the internal part is rather small and does not require a large number of channels.

In contrast, for the $L = 1$ waves ($J^\Pi = 0^-$, 1^-, and 2^-) the convergence rate is slow and many channels have to be included. In these cases, the interaction between the p and ^{3}He clusters is very attractive (it has been speculated that some 4N resonant states exist), and the construction of the internal wave function is more delicate.

An example of convergence for the 0^- and 2^- states is reported in Table I (in the $J = 2$ case the results are relative to the $L = S = 1$ wave). The calculation has been performed using the AV18 potential at $E_{\mathrm{c.m.}} = 1.69$ MeV for a few values of the number N_c of channels included in the expansion of Ψ^J_C. The values $\eta^{J\Pi} \approx 1$ for $N_c = 0$ is accidental, in fact, the value of $\eta^{J\Pi}$ increases after including a few channels more. The convergence is reached only for $N_c \gg 10$ when $|\eta^{J\Pi} - 1| \approx 10^{-5}$.

At the energies considered here, the scattering in the $L = 2$ waves ($J^\Pi = 1^+$, 2^+, and 3^+) is very peripheral and the corresponding phase shifts are small. They can be calculated with good precision by considering only the asymptotic part in Eq. (2). The contribution of the waves with $L > 2$ is very tiny and has been disregarded.

TABLE I. Eigenphase shifts δ^J_{LS} (in degrees) and inelasticity parameters $\eta^{J\Pi}$ for the p-^{3}He scattering waves ${}^3P_0$ and ${}^3P_2$ at $E_{\mathrm{c.m.}} = 1.69$ MeV, calculated with the AV18 potential and the complex Kohn variational method. N_c is the number of channels included in the CHH expansion of the wave functions (the case $N_c = 0$ corresponds to including in the WF only the asymptotic part).

	${}^3P_0$			${}^3P_2$	
N_c	δ^0_{11}	η^{0-}	N_c	δ^2_{11}	η^{2-}
0	3.9	1.0001	0	8.4	1.0000
2	4.2	1.0005	2	8.9	1.0002
4	4.8	1.0021	5	10.0	1.0021
6	6.3	1.0020	10	12.4	1.0005
8	6.5	1.0015	15	12.9	1.0004
9	6.6	1.0002	25	13.3	1.0002
18	6.9	1.0001	35	13.5	1.0001
45	7.0	1.0000	100	13.6	1.0000

The predicted analyzing powers are compared with the measurements in Fig. 1. The solid (dashed) curves correspond to the AV18 (AV18UR) interaction model. The main aspect to be seen in Fig. 1 is that the calculations significantly underpredict the analyzing power by approximately 30% at 1.20 MeV and 40% at 1.69 MeV. Similar results have previously been seen and well documented for N-d scattering. We also see in Fig. 1 that the 3N interaction has almost no effect on A_y.

In Table II the theoretical phase shift parameters at $E_{\mathrm{c.m.}} = 1.20$ MeV are compared with those obtained in the energy-dependent phase shift analysis (PSA) of Ref. [26] which well reproduce the observables shown in Fig. 1. As can be seen, some phase shifts are well reproduced by the theory. However, the ${}^3P_2$ and ${}^3P_1$ phase shifts are both sizably underpredicted by the theory. For some partial waves the errors are too large to make a conclusive statement about the quality of the calculation.

The A_y observable is sensitive mainly to the P-wave phase shifts. At $E_{\mathrm{c.m.}} = 1.20$ MeV, for example, the A_y

TABLE II. S- and P-wave phase shift and mixing angle parameters (in degrees) at $E_{\mathrm{c.m.}} = 1.2$ MeV calculated with the AV18 and the AV18UR potentials. The values obtained with the energy dependent PSA of [26] are also shown.

Wave	AV18	AV18UR	PSA
${}^1S_0$	−33.3	−31.3	−27.4 ± 3.5
${}^3S_1$	−28.8	−27.1	−26.5 ± 0.6
${}^3P_0$	4.1	3.2	2.6 ± 0.6
${}^3P_1$	8.1	7.4	10.1 ± 0.5
${}^3P_2$	7.7	6.9	8.9 ± 0.5
${}^1P_1$	6.5	5.5	4.2 ± 1.5
$\epsilon(1^-)$	−13.9	−13.5	−7.8 ± 0.6

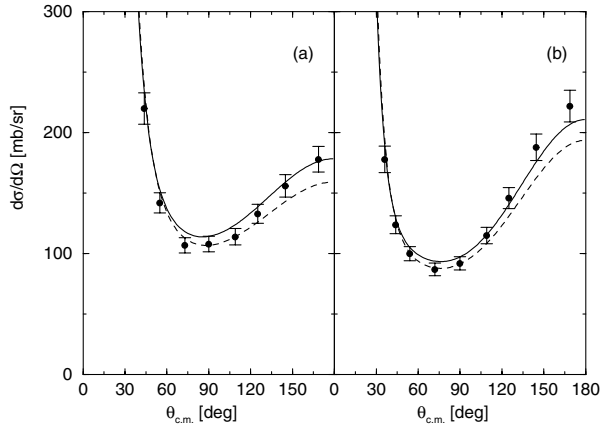

FIG. 2. As in Fig. 1, but for the differential cross section. The data points are from Ref. [27].

maximum increases by approximately 42% when the $^3P_2$ phase shift is changed by +10%. On the other hand, changing the $^3P_0$ ($^3P_1$) phase by +10% decreases the A_y maximum by 8 (6)%. This observable is in particular sensitive to the combination of phase shifts $\Delta = 0.5[\delta(^3P_1) + \delta(^3P_2)] - \delta(^3P_0)$. At $E_{\text{c.m.}} = 1.20$ MeV, the theoretical calculation predicts $\Delta = 3.9°$ (3.8°) with (without) the 3N force. The corresponding experimental result (from the PSA) is $\Delta = 6.9° \pm 0.9°$.

It is interesting to note that the discrepancy between the theoretical and experimental A_y is very much like that for the N-d case. There the main problem is that the splitting between the $^4P_{1/2}$ phase and the average of the $^4P_{3/2}$ and $^4P_{5/2}$ phases is too small. For example, for p-d scattering at $E_{\text{c.m.}} = 2$ MeV, the calculations give $0.5[\delta(^4P_{3/2}) + \delta(^4P_{5/2})] - \delta(^4P_{1/2}) = 1.87°$, whereas the phase shift fits to A_y require a splitting of 2.61° [9]. It is plausible to suspect that the A_y problems for N-d and p-^{3}He both arise from the same deficiency in the nuclear Hamiltonian. Investigations in this direction are actively being pursued.

At the energies studied here old differential cross section data exist [27]. The theoretical estimates are compared with these data in Fig. 2. Overall the agreement is good, especially in the minimum region, but at large angles a small discrepancy is seen. This discrepancy can once again be traced to the underprediction of the calculated $^3P_2$ and $^3P_1$ phase shifts. For this observable the inclusion of the 3N force produces small but non-negligible effects. Finally, there are also some measurements of the ^{3}He analyzing power in the energy range of interest [28]. For this observable, the theoretical estimates are again below the data, but unfortunately the experimental uncertainties are too large to make a definitive statement.

In summary, we have performed calculations of low-energy p-^{3}He scattering observables, employing a realistic interaction with a 3N force, and including the effects of P and D waves. A comparison with new proton analyzing power measurements demonstrates that the A_y puzzle also exists in the 4N system.

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