Optical Instruments

1) INTRODUCTION

In Chapter 26 of the text we learned how lenses and mirrors can be used to form images. We will now look at some of the real-life applications of this technology. In particular the goal is to understand how distant or very small objects can be seen more clearly with the aid of a telescope or microscope. Understanding how these instruments work requires that we first know a little about how images are formed in our eyes.

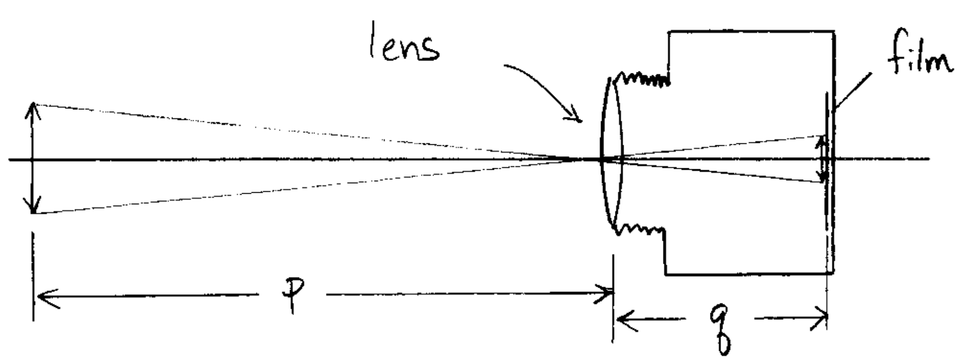
2) A SIMPLE CAMERA

Our eyes are similar in some respects to a camera. A simple camera can consist of a single converging lens which is used to form a real image on film located at the back of the camera. Lets suppose that the object we want to photograph is located a distance p in front of the lens. As usual, the image location can be found from the formula

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

which, solving for the image location, gives

$$q = \frac{pf}{p-f}.$$



Let's see what the image distances turn out to be if our camera is fitted with a lens that has a focal length of 50 mm. To take a closeup picture of a flower at, for example, $p = 50\,\text{cm}$ we find that the image is at $q = 55.5\,\text{mm}$. On the other hand, for distant objects $p >> f$ and we find $q \simeq f$. So the conclusions are that we need lens-to-film distances in the range 50-56 mm, and that we can obtain sharply focussed images (for objects between 50 cm and infinity) by moving the lens only a few millimeters.

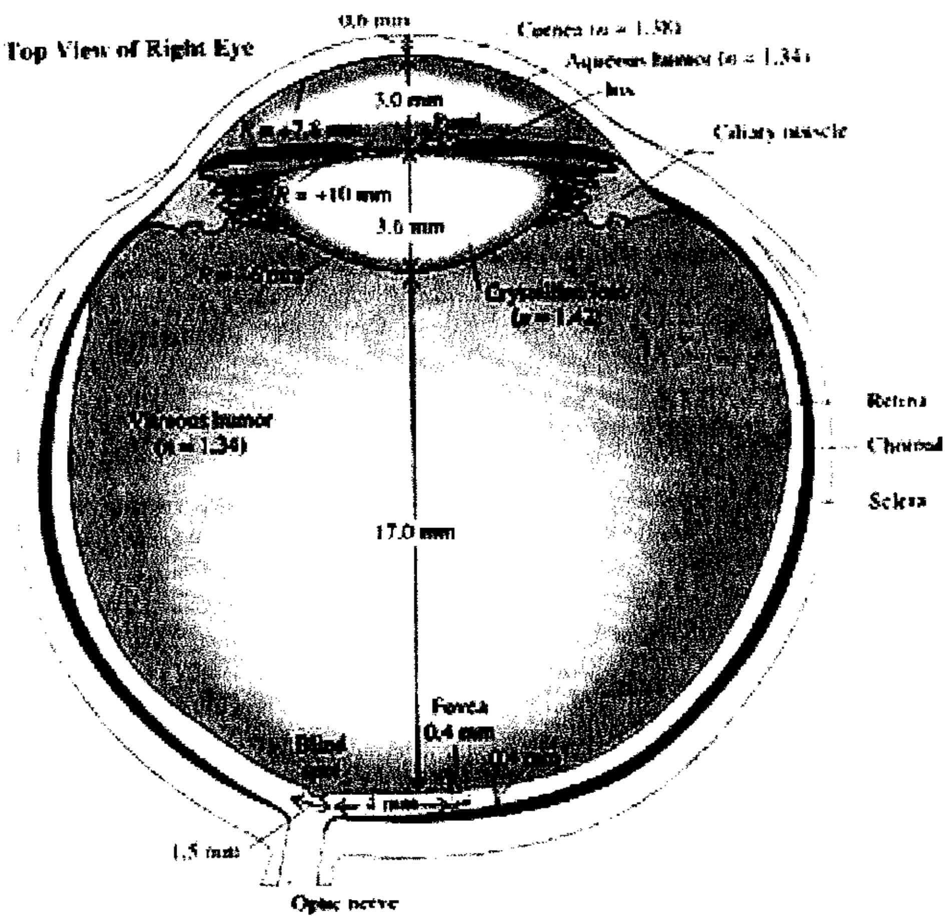
You can also easily find the image size on the film by using the magnification formula,

$$\frac{h'}{h} = -\frac{q}{p}.$$

The image is always inverted, and if you take a picture, for example, of a 2 m tall basketball player standing 5 m away, the image will turn out to be 19.8 mm high, covering a bit over half the width of standard 35 mm film.

2) EYES

The drawing below shows the basic structure of the human eye. Light entering the eye is focussed by the curved outer surface of the cornea working together with the crystalling lens. An image is formed on the retina, the back surface of the eye, which is made up of nerve fibers and light sensitive cells. The distance from the front of the cornea to the retina is around 25 mm, so the focal length of the cornea/lens system needs to be in that general range.



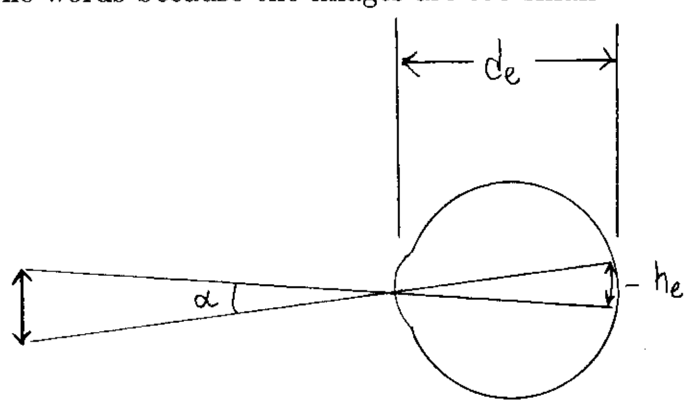
In a camera, focussing is accomplished by moving the lens with respect to the film, but the eye works in a different way. Here images are brought into focus with the ciliary muscle, which stretchs the crystalling lens, changing it's focal length. The muscle relaxes to view distant objects, and contracts for closeup vision (e.g. reading).

A given individual will have a field of clear, focussed vision that ranges from some "far point", x_f, to some "near point", x_n. For "normal" vision the far point is infinity, and the near point is around 20-30 cm. As we age, our crystalline lenses grow thicker and thicker, and the consequence is that x_n gradually increases, eventually making it impossible to read without glasses. In contrast, infants may have a near point of 10 cm or less.

From the lens formula we can find the range of focal lengths needed for normal vision. In the eye the lens-to-retina distance, d_e, is fixed at about 25 mm, and so we set $q = d_e = 25\,\mathrm{mm}$. For viewing distant objects we have $p = x_f = \infty$ which gives $f = d_e = 25\,\mathrm{mm}$. At the opposite extreme, for closeup viewing we take $p = x_n = 25\,\mathrm{cm}$, and find that the required focal length is $f = 22.7\,\mathrm{mm}$.

The images on the retina are not very large. The printed letters in a textbook are around 2 mm high, so if you hold the book at 30 cm, the images are around $\frac{1}{6}$ mm in size. While the image is small, we can still make out enough detail to distinguish one letter from another because the $\frac{1}{6}\,\mathrm{mm} \times \frac{1}{6}\,\mathrm{mm}$ spot on your retina has several thousand light sensitive cells. If you hold the book at 2 m instead of 30 cm, you can no longer read the words because the images are too small.

Notice that because the image distance is fixed at d_e, the height of the image on the retina, h_e, can be expressed in a new way in terms of the angle α between rays arriving at the eye from the top and from the bottom of the object. As we see from the drawing



$$h_e = \alpha d_e,$$

where α is to be expressed in radians.

3) EYEGLASSES

Vision problems can often be corrected with eyeglasses. Many people are either "nearsighted" or "farsighted", conditions that arise if the cornea is either too flat or too curved. If your cornea has a bit too much curvature, light rays are focussed more than they should be and the result is that distant objects will be blurred. Your far point, x_f, might be a meter or two (instead of infinity), and your near point distance would ordinarily be less than the "normal" 25 cm. A person with this condition is said to be nearsighted. Farsightedness occurs when the cornea is a bit too flat. In this case your distance vision will be adequate, but your eyes will not have enough power to focus on objects that are held close. In other words, x_n may be significantly greater than 25 cm.

The conditions described above can easily be corrected with simple converging or diverging lenses. Lenses are useful when they produce an upright, virtual image located on the front side of the lens, i.e. on the same side as the object. When you look through the lens, it is the virtual image that your eye sees, and your eye will be able to bring the image into focus provided that the image location, x, is within your range of vision, $x_n \leq x \leq x_f$.

To illustrate the idea, lets consider how to correct the vision of someone who is nearsighted. If the individual's far point, x_f, is 1 or 2 m, objects like highway signs will be blurred. We need to choose a lens that will take distant objects (with p up to infinity) and produce virtual images in the range of vision $x \leq x_f$. In particular, the goal is to arrange it so that objects at $p = \infty$ give images at $x = x_f$. For a virtual image, q is negative, and from the drawing below we see that the distance from the eye to the virtual image will be

$$x = -q + \Delta x$$

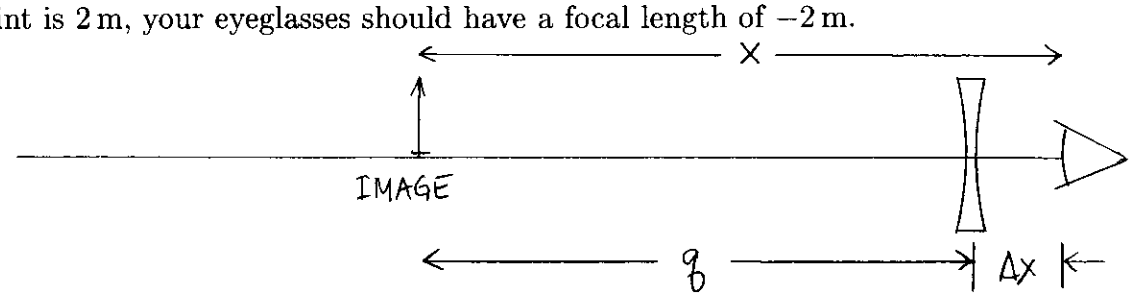
where Δx is the distance between the lens and the eye (zero for contact lenses and 1-2 cm for glasses). For simplicity we ignore the Δx, and it follows that what we want is

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} + \frac{1}{-x_f}$$

which gives

$$f = -x_f.$$

If your far point is 2 m, your eyeglasses should have a focal length of -2 m.



Notice that in the above example objects at large but finite distances will be within your range of vision. For example $p = 5\,\mathrm{m}$ gives $q = -1.43\,\mathrm{m}$ and $x = 1.43\,\mathrm{m}$ which is fine.

The general ideas outlined above can also be applied to farsightedness. If the letters are blurred when you try to read, you need lenses that convert the objects (the letters) into virtual images that are farther from your eyes, in the range $x \geq x_n$. It is easy to demonstrate that this can be done with converging lenses.

4) THE MAGNIFYING GLASS

The ability to see an object clearly requires that two conditions be met. First of all, the image formed on the retina must be in focus, or at least nearly so, and second, the image size, h_e, needs to be large enough for us to make out the important details.

When you look at small objects, the best you can do with your unaided eye is to hold the object at your near point. This leads to the largest possible value of α,

$$\alpha_0 \simeq \frac{h}{x_n},$$

and consequently the largest h_e, consistent with the requirement of focusing.

A magnifiying glass is a simple converging lens that, when used correctly, allows us to increase h_e. The ideas are similar to those we used in the previous section. The goal is to produce a virtual image, and when the magnifying glass is held in front of your eye, it is this virtual image that you see. It follows that the properties of the virtual image – it's size and distance from your eye – determine how the object will look to you.

Lets assume that the lens has a focal length f, and that you place the object inside the focal point, $p < f$. From the lens equation

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p},$$

we see that q will be negative, which means that the image is virtual and upright. Since our goal is to enlarge the image on the **retina**, the relevant quantity is the **angular size** of the virtual image,

$$\alpha \simeq \frac{h'}{x},$$

where x is the distance of the image from your eye.

Lets suppose you hold the magnifying glass directly in front of your eye to make Δx negligible. Then the image will be a distance $x = -q$ from your eye and we obtain

$$\alpha \simeq \frac{h'}{-q} = \frac{h}{p} = h\left[\frac{1}{f} - \frac{1}{q}\right] = h\left[\frac{1}{f} + \frac{1}{x}\right].$$

Since the image size on the retina, h_e, is proportional to α, we define the **magnifying power** of the lens to be

$$m = \frac{h_e(\text{magnified})}{h_e(\text{unaided})} = \frac{\alpha}{\alpha_0} = x_n\left[\frac{1}{f} + \frac{1}{x}\right].$$

This quantity is sometimes called the "angular magnification" of the lens.

According to this last equation, the magnifying power depends on x which is the distance of the virtual image from your eye. As you use the magnifier, you need to adjust the object distance until the virtual image can be focussed by your eye, and this means that x needs to be somewhere in the range $x_n \leq x \leq x_f$. Since x_f is normally infinite, we conclude that the magnification will be in the range

$$\frac{x_n}{f} \leq m \leq \frac{x_n}{f} + 1.$$

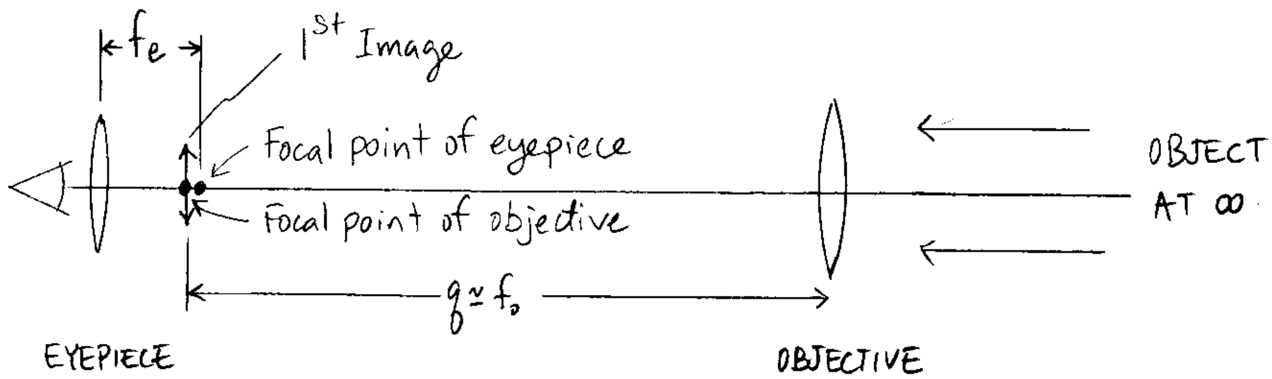
Taking $x_n = 25\,\text{cm}$ we conclude that a lens with $f = 5\,\text{cm}$ will have a magnifying power of 5-6. In subsequent applications we will use the lower limit, $m = x_n/f$.

5) MICROSCOPES AND TELESCOPES

As you know, our eyes are not capable of resolving any detailed visual information for objects that are too small (cells) or too distant (planets). In both cases the problem is simply that the image formed on the retina of our eye is too small. Microscopes and telescopes use lenses, or combinations of lenses and mirrors to magnify these images.

In this section we will analyze a simple microscope and a simple telescope. The two instruments have a number of features in common. Both use a simple design with two converging lenses. The first lens, called the objective, is used to produce a real image, while the second lens, called the eyepiece, is really just a magnifying glass which we adjust to view the first image.

Lets discuss the telescope first. Suppose that the objective lens has a focal length f_o. If we are looking at a planet then the object distance $p = d$ will be very large compared to f_o, and we conclude that first image will be essentially at the focal point of the lens, $q = f_o$. We now view this real image using the eyepiece lens as a magnifying glass. Recall that this second image needs



to be located at some moderate to large distance in front of your eye, and this requires that the first image be located just inside the focal point of the eyepiece, as illustrated in the drawing.

To determine the magnifying power of the telescope we need to consider the image sizes. The size of the first image, h_1, is given in terms of the object size, h_o, by the magnification formula,

$$h_1 = -\frac{f_o}{d}\, h_o.$$

Suppose we were to view this image directly with our eyes, instead of with the eyepiece. We would need the image to be at our nearpoint, and so the angular size would be

$$\alpha_1 \simeq \frac{h_1}{x_n} = -\frac{1}{x_n}\frac{f_o}{d}\, h_o.$$

From the previous section we know that by using the eyepiece instead of our eye, we will gain an additional factor of x_n/f_e and so we conclude that the angular size of the second image will be

$$\alpha_2 = -\frac{f_o}{f_e}\frac{h_o}{d}.$$

The magnifying power of the telescope is defined to be the **angular size of the final image** divided by the **angular size of the object** as we would see it with the unaided eye. This last quantity is just

$$\alpha_0 = \frac{h_o}{d},$$

and so we have

$$m = \frac{\alpha_2}{\alpha_0} = -\frac{f_o}{f_e}.$$

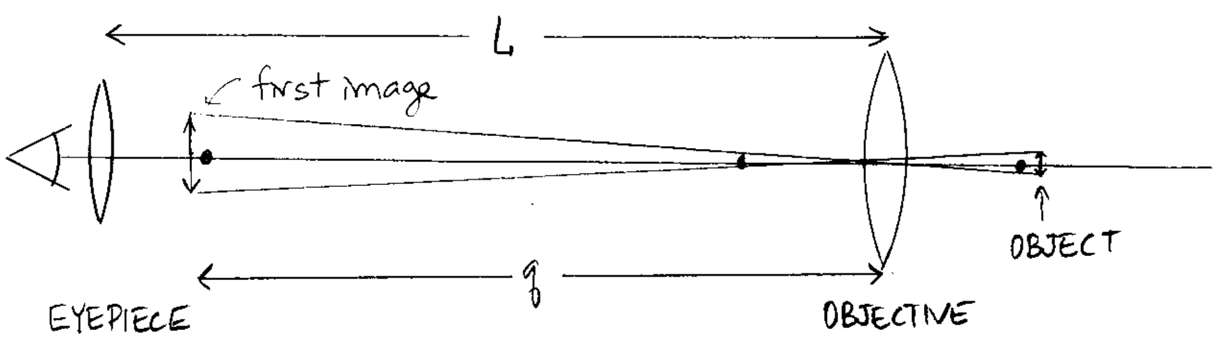
The magnifying power of a telescope is an important parameter, but in many applications there is another issue. If we want to look at distant stars and galaxies, the total amount of light per unit area arriving from the object can be quite small, and for this reason it is desirable to use a large objective lens to collect as much light as possible. This along with the fact that the magnifying power is proportional to f_o, and that the overall length of the telescope is $f_o + f_e$, means that astronomical telescopes are normally quite large.

The drawing below shows a simple compound microscope. As before, the objective lens produces a real image which is supposed to be located at or just inside the focal point of the eyepiece. To maximize the size of the first image we want q/p to be as large as possible, and this is accomplished by using an objective lens with a relatively short focal length and placing the object just **outside** the focal point of that lens. The magnification of the objective is

$$M = -\frac{q}{p} = -q\left[\frac{1}{f_o} - \frac{1}{q}\right] = -\left[\frac{q}{f_o} - 1\right] = -\frac{q - f_o}{f_o}.$$

If L is the overall length of the microscope (as defined in the drawing) then $q \simeq L - f_e$ and so the size of the first image can be expressed as

$$h_1 = -\frac{L - f_o - f_e}{f_o}\, h_o.$$



As before, suppose we were to view this image with our eye. The angular size would be

$$\alpha_1 \simeq \frac{h_1}{x_n} = -\frac{1}{x_n}\,\frac{L - f_o - f_e}{f_o}\, h_o,$$

and so with the additional magnification of the eyepiece we have

$$\alpha_2 = -\frac{L - f_o - f_e}{f_o f_e}\, h_o.$$

Finally, to obtain the magnifying power of the instrument we need to compare α_2 with the angular size of the original **object** as we would see it with the unaided eye. That quantity is just $\alpha_0 \simeq h_o/x_n$ and so we obtain

$$m = \frac{x_n(L - f_o - f_e)}{f_o f_e}.$$