

1) (a) List the possible values of n , l , and m for an electron in the 4p state.

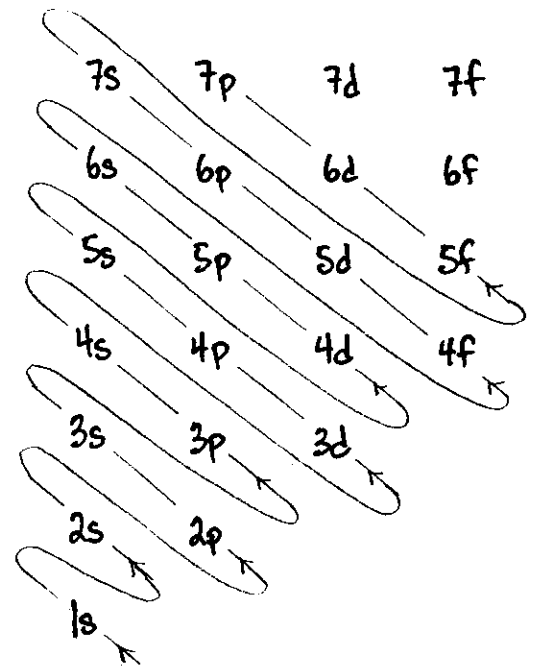
$$\boxed{n=4}$$

$$\boxed{l=1} \quad \boxed{m=0, \pm 1}$$

(b) What are the possible values of the angular momentum, L , for an electron with $n = 3$.

$$\boxed{L=0, \sqrt{2} \hbar, \sqrt{6} \hbar}$$

(c) Except for helium, the inert gasses are elements with just enough electrons to fill the outermost p-shell. Assuming that the filling order is given by the drawing at the right, find element number (Z) of the first 5 inert gas elements beyond helium.



	$(1s)^2$	$(2s)^2$	$(2p)^6$	$(3s)^2$	$(3p)^6$	$(4s)^2$	$(3d)^{10}$	$(4p)^6$
e's	2	2	6	2	6	2	10	6
Total	2	4	$\boxed{10}$	12	$\boxed{18}$	20	30	$\boxed{36}$
	$(5s)^2$	$(4d)^{10}$	$(5p)^6$	$(6s)^2$	$(4f)^{14}$	$(5d)^{10}$	$(6p)^6$	
	2	10	6	2	14	10	6	
	38	48	$\boxed{54}$	56	70	80	$\boxed{86}$	

Answer: 10, 18, 36, 54, 86

2) An electron is confined in a one-dimensional square well potential 0.5 nm wide. Find the wavelength of the photon that is emitted if the electron jumps from the first excited state to the ground state. The electron's mass is 9.11×10^{-31} kg which corresponds to $mc^2 = 5.11 \times 10^5$ eV.

$$\bar{E}_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} \Rightarrow E_\gamma = E_2 - E_1 = 3 \frac{\pi^2 \hbar^2}{2mL^2} = \frac{hc}{\lambda}$$

$$\lambda = \frac{(hc)(2mL^2)}{3\pi^2 \hbar^2} = \frac{(hc)(8mL^2)}{3(2\pi\hbar)^2} = \frac{8}{3} \frac{mc^2 L^2}{hc}$$

$$= \frac{8}{3} \frac{(5.11 \times 10^5 \text{ eV})(0.5 \text{ nm})^2}{1240 \text{ eV} \cdot \text{nm}} = 275 \text{ nm}$$

$$\boxed{275 \text{ nm}}$$

3) Suppose your eyes are able to focus objects that are between 20 cm and 50 cm. You would like contact lenses that permit you to clearly see things that are very far away.

(a) What focal length should the lenses have?

For an object at $s = \infty$ we want the image at the far point $\Rightarrow s' = -50 \text{ cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-50 \text{ cm}} = \frac{1}{f}$$

$$f = -50 \text{ cm.}$$

(b) If you wear your lenses while reading, how far away do you need to hold the book in order to see the words clearly.

s needs to be large enough to get the image beyond -20 cm

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s} = \left(\frac{1}{-50 \text{ cm}}\right) - \left(\frac{1}{-20 \text{ cm}}\right) \Rightarrow s = 33.3 \text{ cm}$$

$$33.3 \text{ cm}$$

4) π -mesons normally decay into a muon and a neutrino. When the π is at rest, the muon is always emitted with a velocity of $0.28c$. Find the velocity of the muon emitted in the decay of a moving π ; assume that the π 's velocity is $0.8c$ and that the muon is emitted along the direction of motion.

Let S be the "lab" frame and S' be a frame moving with the π . So in S' the π is at rest and the muon will have velocity $0.28c$. We have

$$u_x' = \text{vel. of the object} = 0.28c$$

$$v = \text{" of } S' \text{ relative to } S = 0.80c$$

$$u_x' = \frac{u_x - v}{1 - vu_x/c^2} \Rightarrow u_x'(1 - vu_x/c^2) = u_x - v$$

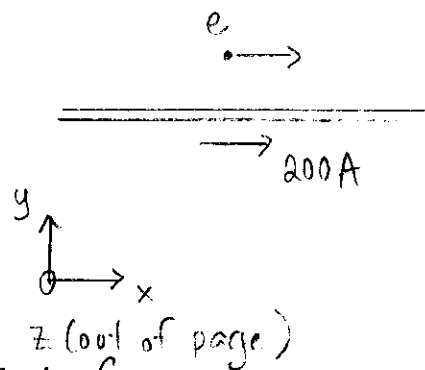
$$u_x' - vu_x u_x'/c^2 = u_x - v \quad \text{Solve for } u_x$$

$$u_x \left(1 + \frac{vu_x'}{c^2}\right) = u_x' + v$$

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{0.28c + 0.80c}{1 + (0.28)(0.80)}$$

$$0.882c$$

- 5) An electron is 5 cm from an infinitely long wire carrying a current of 200 A. Find the magnitude and direction of the force acting on the electron if its kinetic energy is 3×10^5 eV. The electron mass is given in Problem 2.



$$F = q \vec{v} \times \vec{B}$$

Find v from the kinetic energy. Notice that if you use $KE = \frac{1}{2}mv^2$ you find $v > c$ which means that we need to use the relativistic formula

$$KE = (\gamma - 1)m_0c^2 \quad (\gamma - 1) = \frac{KE}{m_0c^2} = \frac{3 \times 10^5 \text{ eV}}{5.11 \times 10^5 \text{ eV}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.587 \Rightarrow v = 0.777c = 2.33 \times 10^8 \text{ m/s}$$

$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(200 \text{ A})}{(2\pi)(0.05 \text{ m})} = 8 \times 10^{-4} \text{ T}$$

$$|\vec{F}| = qvB = (1.6 \times 10^{-19} \text{ C})(2.33 \times 10^8 \text{ m/s})(8 \times 10^{-4} \text{ T}) = 2.98 \times 10^{-14} \text{ N}$$

DIRECTION: \vec{B} is out of the page

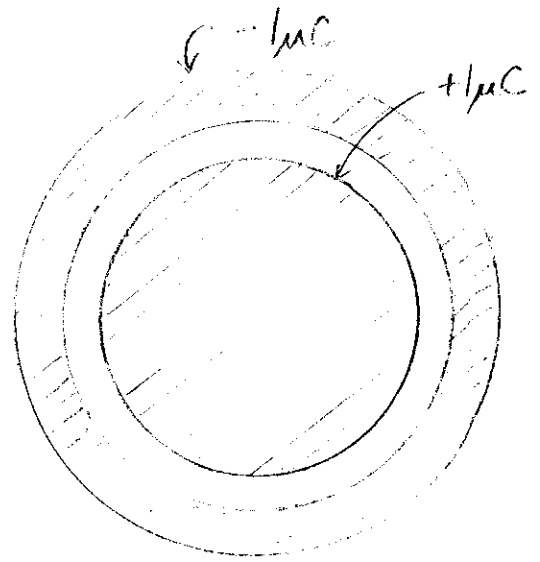
$\vec{v} \times \vec{B}$ is downward (-y)

q is negative $\Rightarrow \vec{F}$ is along +y

Magnitude: $2.98 \times 10^{-14} \text{ N}$

Direction: +y

- 6) A conducting sphere of radius 20 cm carries a net charge of $1 \mu\text{C}$. Around the sphere is a conducting spherical shell with inner radius of 22 cm and outer radius 25 cm. The shell carries a net charge of $-1 \mu\text{C}$.



- (a) Find the electric field at the surface of the sphere.

Field is the same as a point charge

$$E = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r^2}$$

$$= (10^{-6}\text{C})(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}) \left(\frac{1}{0.20\text{m}}\right)^2 = 2.25 \times 10^5 \text{ N/C}$$

$$2.25 \times 10^5 \text{ N/C}$$

- (b) Approximately what is the voltage difference between the two conductors?

The field is approximately constant between the two conductors, so

$$V \approx E \cdot d = (2.25 \times 10^5 \frac{\text{N}}{\text{C}})(0.02\text{m}) = 4500 \text{ volts.}$$

$$4500 \text{ volts}$$

The average field is more like the field @ $r = 21 \text{ cm}$ which

$$\text{is } (10^{-6}\text{C})(8.99 \times 10^9) \left(\frac{1}{0.21\text{m}}\right)^2 = 2.04 \times 10^5 \text{ V/m}$$

$$\text{giving } \Delta V = 4080 \text{ volts.}$$

7) For any state with $\ell = 0$ the Schrodinger equation for hydrogen can be written in the form

$$-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} [r\psi(r)] + U(r) [r\psi(r)] = E [r\psi(r)].$$

where

$$U(r) = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{r}$$

Show that $\psi = Ce^{-\lambda r}$ solves the equation for the right value of λ . Find the correct λ and the energy of the corresponding state. Your results should be given as formulas. Work must be shown to get credit.

$$r\psi = Cre^{-\lambda r}$$

$$\frac{d}{dr} (r\psi) = C [e^{-\lambda r} - \lambda r e^{-\lambda r}]$$

$$\begin{aligned} \frac{d^2}{dr^2} (r\psi) &= C [-\lambda e^{-\lambda r} - \lambda e^{-\lambda r} + \lambda^2 r e^{-\lambda r}] \\ &= C [-2\lambda + \lambda^2 r] e^{-\lambda r} \end{aligned}$$

Now plug into the equation - work on the left-hand side

$$-\frac{\hbar^2}{2m} [C (-2\lambda + \lambda^2 r)] e^{-\lambda r} - \frac{e^2}{4\pi\epsilon_0} \frac{1}{r} r C e^{-\lambda r}$$

Factor out $r C e^{-\lambda r}$

$$r C e^{-\lambda r} \left[\frac{\hbar^2}{m} \cdot \frac{\lambda}{r} - \frac{\hbar^2 \lambda^2}{2m} - \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r} \right] = E r \psi(r)$$

For the equation to have the right form (constant $\times r \psi(r)$)

we need $[\]$ to be a constant \Rightarrow the $\frac{1}{r}$ terms must cancel.

$$\Rightarrow \frac{\hbar^2}{m} \lambda - \frac{e^2}{4\pi\epsilon_0} = 0 \quad \lambda = \frac{e^2}{4\pi\epsilon_0} \cdot \frac{m}{\hbar^2}$$

Then we have left

$$E = -\frac{\hbar^2 \lambda^2}{2m} = -\frac{\hbar^2}{2m} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \left(\frac{m}{\hbar^2} \right)^2$$

$$\lambda: \boxed{\frac{e^2}{4\pi\epsilon_0} \cdot \frac{m}{\hbar^2}}$$

$$E: \boxed{-\frac{m}{2} \left(\frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{\hbar} \right)^2}$$

- 8) (a) The isotope ^{15}O has a half-life of 120s, and an atomic mass of 15 u. Find the activity of a 1 μg sample of ^{15}O . 1 mole would have 15 grams, so we have

$$(6.02 \times 10^{23}) \left(\frac{10^{-6} \text{g}}{15 \text{g}} \right) = 4.01 \times 10^{16} \text{ nuclei}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \ln 2 / 120 \text{s} = 5.8 \times 10^{-3} / \text{s}$$

$$R = \lambda N = 2.3 \times 10^{14} / \text{s}$$

$$2.3 \times 10^{14} / \text{s}$$

- (b) Find the activity of the sample after 5 minutes.

$t_{\frac{1}{2}} = 2 \text{ min}$, so we have 2.5 half-lives

$$R = R_0 \left(\frac{1}{2} \right)^{2.5} = 0.177 R_0$$

$$4.1 \times 10^{13} / \text{s}$$

- 9) In the circuit shown at the right, the switch is initially open and the capacitor is uncharged.

- (a) Find the current in the 100 Ω resistor the instant after the switch is closed.

Initially there is no voltage across C, so there must be 12 volts across 100 Ω

$$I = \frac{V}{R} = \frac{12 \text{V}}{100 \Omega}$$

$$0.12 \text{ A}$$

- (b) Find the current in the 100 Ω resistor after the switch has been closed for a long time.

The capacitor is now fully charged so the only current path is through the 200 Ω resistor

$$I = \frac{12 \text{V}}{300 \Omega} = 0.040 \text{ A}$$

$$0.040 \text{ A}$$

