RELATIVITY HOMEWORK

3) The elapsed time measured on the space probe is $\Delta t = 3.0s$. This is the proper time interval, so as seen from the earth

$$\Delta t = \gamma \Delta t_p \qquad \gamma = \left[\frac{1}{1-v^2/c^2}\right]^{\frac{1}{2}} = \left[\frac{1}{1-(0.8)^2}\right]^{\frac{1}{2}} = 1.667$$

$$\Delta t = (1.667)(3s) = \boxed{5s}$$

5) Here $\gamma = \left[\frac{1}{1-(.98)^2}\right]^{\frac{1}{2}} = 5.025$

(a) $\Delta t = \gamma \Delta t_p = \boxed{13.1 \times 10^{-8} s}$

(b) distance $= s = v\Delta t = (0.98)(3\times10^8 m/s)(13.1\times10^{-8}s)$

$$\boxed{s = 38.4m}$$

(c) With no time dilation the lifetime would be $2.6\times10^{-8}s$

$\Rightarrow$

$$s = (0.98)(3\times10^8 m/s)(2.6\times10^{-8}s) = \boxed{7.64m}$$

6) At $v = 0.95c$ $\quad \gamma = 3.20$

(a) The time is reduced by a factor of γ, because the astronauts time is the proper time $\Rightarrow$

$$t = (4.42 \text{ years})/\gamma = \boxed{1.38 \text{ years}}$$

(b) The distance is shortened by γ

$$d = (4.2 \text{ light years})/\gamma = \boxed{1.31 \text{ light-yrs}}$$

15) The electron's momentum is $p_e = \gamma_e m_e v_e$ and we want the proton to have an equal momentum:

$$p_p = \gamma_p m_p v_p = \gamma_e m_e v_e$$

where

$$\gamma = \left[\frac{1}{1-v^2/c^2}\right]^{\frac{1}{2}}$$

To solve for v_p we need to square and re-arrange

$$\frac{m_p^2 v_p^2}{1 - v_p^2/c^2} = \gamma_e^2 m_e^2 v_e^2$$

$$\left(\frac{m_p}{m_e}\right)^2 v_p^2 = \gamma_e^2 v_e^2 \left(1 - \frac{v_p^2}{c^2}\right)$$

$$v_p^2 \left[\left(\frac{m_p}{m_e}\right)^2 + \gamma_e^2 \frac{v_e^2}{c^2} \right] = \gamma_e^2 v_e^2$$

$$v_p = \frac{\gamma_e v_e}{\left[\left(\frac{m_p}{m_e}\right)^2 + \left(\gamma_e \frac{v_e}{c}\right)^2 \right]^{1/2}} \qquad \gamma_e = 2.294$$

$$\frac{m_p}{m_e} = 1835$$

$\Rightarrow$

$$\left(\frac{v_p}{c}\right) = (2.294)(0.9) / \left[(1835)^2 + (2.294)^2 (0.9)^2 \right]^{\frac{1}{2}}$$

$$\boxed{v_p = 0.00113\, c}$$

18) The two particles must have equal and opposite momenta. $\Rightarrow$ $p_1 = p_2$

$$\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2$$

We can solve for v_1 just as in the previous problem

$$v_1 = \frac{\gamma_2 v_2}{\left[\left(\frac{m_1}{m_2}\right)^2 + \left(\gamma_2 \frac{v_2}{c}\right)^2 \right]^{\frac{1}{2}}}$$

For $v_2 = 0.893\, c$ $\qquad \gamma_2 = 2.222 \qquad \gamma_2 v_2 = 1.984\, c$

$$\frac{v_1}{c} = \frac{(1.984)}{\left[\left(\frac{1.67}{0.25}\right)^2 + (1.984)^2 \right]^{\frac{1}{2}}} = 0.285 \qquad \boxed{v_1 = 0.285\, c}$$

22) Let S' be a frame moving with the spaceship, and then the velocity of the particle in that frame will be $u' = 0.90c$. If S is the frame of the external observer, then v = velocity of S' relative to S is $v = 0.75c$ The velocities are related by

$$u' = \frac{u - v}{1 - vu/c^2}$$

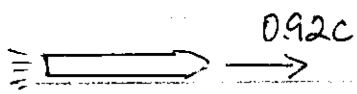
$$u'\left(1 - \frac{vu}{c^2}\right) = u - v$$

$$u' - \frac{vuu'}{c^2} = u - v$$

$$u' + v = u\left[1 + \frac{vu'}{c^2}\right] \Rightarrow u = \frac{u' + v}{1 + vu'/c^2}$$

$$u = \frac{(0.9c) + (0.75c)}{1 + (0.75)(0.9)} = \boxed{0.985c}$$

23) There are many ways



to do this with the velocity formula. We could let S be the frame of observer A and S' be the frame of observer B. Then the speed of the rocket is $u = +0.92c$ according to A and $u' = -0.95c$ according to B. Then solve for v = vel. of B relative to A.

$$u' = \frac{u - v}{1 - vu/c^2} \Rightarrow u' - \frac{vuu'}{c^2} = u - v$$

$$v\left[1 - \frac{uu'}{c^2}\right] = u - u'$$

$$v = \frac{u - u'}{1 - uu'/c^2} = \frac{(0.92c) - (-0.95c)}{1 - (0.92)(-0.95)} = \frac{1.870}{1.874} = \boxed{0.998c}$$

26) $v = 0.95c \Rightarrow \gamma = 3.2$ $\qquad m_pc^2 = 938.27\text{ MeV}$ (from book)

(a) Rest energy $= m_pc^2 = \boxed{938.27\text{ MeV}}$

(b) Total " $= \gamma m_pc^2 = (3.2)(938.27\text{ MeV}) = \boxed{3004.87\text{ MeV}}$

(c) $KE = (\gamma - 1)m_pc^2 = \boxed{2066.6\text{ MeV}}$

27) (a) $E = mc^2 = (0.5\text{ kg})(3\times10^8\text{ m/s})^2 = \boxed{4.5\times10^{16}\text{ J}}$

(b) $100\text{ W} = 100\text{ J/s}$

$$\Delta t = E/P = (4.5\times10^{16}\text{ J})/(100\text{ J/s}) = \boxed{4.5\times10^{14}\text{ sec}}$$
$$= \boxed{1.4\times10^7\text{ years} = 14\text{ billion yrs}}$$

31) $KE = q\cdot\Delta V = (1.6\times10^{-19}\text{ C})(20{,}000\text{ V}) = 20{,}000\text{ eV}$

But

$KE = (\gamma - 1)m_ec^2$ $\qquad m_ec^2 = 5.11\times10^5\text{ eV}$

$$\gamma = 1 + \frac{20{,}000}{511{,}000} = 1.039 = \left[\frac{1}{1-(v/c)^2}\right]^{\frac{1}{2}}$$

Solving for v gives

$$\boxed{v = 0.272c}$$

33) Energy conservation $\Rightarrow \gamma_1 m_1 \cancel{c^2} + \gamma_2 m_2 \cancel{c^2} = m_0 \cancel{c^2}$

where m_0 = rest mass of original particle $= 3.34\times10^{-27}$ kg.

1 and 2 have equal & opposite momenta

$$\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2 \Rightarrow \gamma_2 m_2 = \left(\frac{v_1}{v_2}\right)\gamma_1 m_1$$

$\Rightarrow$

$$\gamma_1 m_1 + \gamma_1 m_1\left(\frac{v_1}{v_2}\right) = m_0 \Rightarrow m_1 = \frac{m_0}{\gamma_1(1 + v_1/v_2)} = \frac{3.34\times10^{-27}\text{ kg}}{(6.22)(1 + .987/.868)}$$

$$m_1 = \frac{m_0}{13.3} = \boxed{2.51\times10^{-28}\text{ kg}} \qquad m_2 = \frac{m_0 - \gamma_1 m_1}{\gamma_2} = \boxed{8.82\times10^{-28}\text{ kg}}$$