***20.30** Substituting given values into $V = \frac{k_e q}{r}$,

$$7.50 \times 10^3 \text{ V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)q}{0.300 \text{ m}}$$

Substituting $q = 2.50 \times 10^{-7}$ C,

$$N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C/e}^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$$

20.33 (a) $Q = C\Delta V = \left(4.00 \times 10^{-6} \text{ F}\right)(12.0 \text{ V}) = 4.80 \times 10^{-5} \text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = \left(4.00 \times 10^{-6} \text{ F}\right)(1.50 \text{ V}) = 6.00 \times 10^{-6} \text{ C} = \boxed{6.00\ \mu\text{C}}$

20.35 (a) $\Delta V = Ed$

$$E = \frac{20.0 \text{ V}}{1.80 \times 10^{-3} \text{ m}} = \boxed{11.1 \text{ kV/m}}$$

(b) $E = \frac{\sigma}{\epsilon_0}$

$$\sigma = \left(1.11 \times 10^4 \text{ N/C}\right)\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right) = \boxed{98.3 \text{ nC/m}^2}$$

(c) $C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)\left(7.60 \text{ cm}^2\right)(1.00 \text{ m}/100 \text{ cm})^2}{1.80 \times 10^{-3} \text{ m}} = \boxed{3.74 \text{ pF}}$

(d) $\Delta V = \frac{Q}{C}$

$$Q = (20.0 \text{ V})\left(3.74 \times 10^{-12} \text{ F}\right) = \boxed{74.7 \text{ pC}}$$

***20.36** $E = \frac{k_e q}{r^2}$:

$$q = \frac{\left(4.80 \times 10^4 \text{ N/C}\right)(0.210 \text{ m})^2}{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)} = 0.240\ \mu\text{C}$$

(a) $\sigma = \frac{q}{A} = \frac{0.240 \times 10^{-6}}{4\pi(0.120)^2} = \boxed{1.33\ \mu\text{C/m}^2}$

(b) $C = 4\pi\epsilon_0 r = 4\pi\left(8.85 \times 10^{-12}\right)(0.120) = \boxed{13.3 \text{ pF}}$

20.38 (a) $C = \dfrac{\ell}{2k_e \ln\left(\dfrac{b}{a}\right)} = \dfrac{50.0}{2\left(8.99\times10^{9}\right)\ln\left(\dfrac{7.27}{2.58}\right)} = \boxed{2.68\text{ nF}}$

(b) Method 1: $\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$

$$\lambda = q/\ell = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2\left(8.99\times10^{9}\right)\left(1.62\times10^{-7}\right)\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02\text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10\times10^{-6}}{2.68\times10^{-9}} = \boxed{3.02\text{ kV}}$

20.41 (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$$

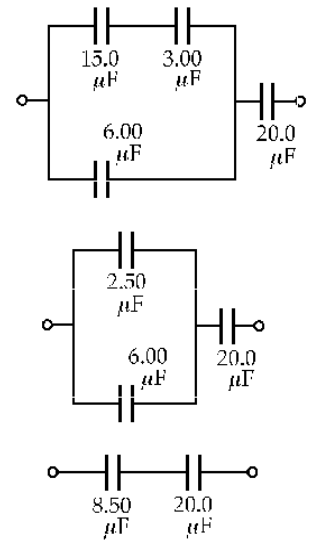
(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00\text{ V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\text{ V}) = \boxed{45.0\ \mu\text{C}}$

and $Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\text{ V}) = \boxed{108\ \mu\text{C}}$

20.43 (a) $\dfrac{1}{C_s} = \dfrac{1}{15.0} + \dfrac{1}{3.00}$



$C_s = 2.50\ \mu\text{F}$

$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$

$C_{eq} = \left(\dfrac{1}{8.50\ \mu\text{F}} + \dfrac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$

(b) $Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

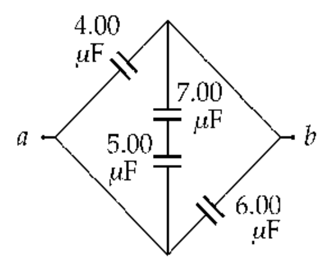
$\Delta V = \dfrac{Q}{C} = \dfrac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$

$15.0 - 4.47 = 10.53\ \text{V}$

$Q = C\Delta V = (6.00\ \mu\text{F})(10.53\ \text{V}) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$

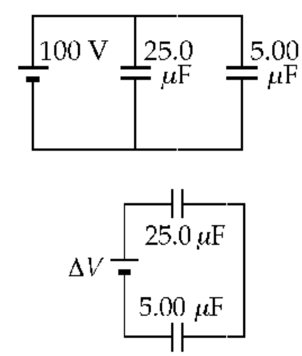
***20.46** $C_s = \left(\dfrac{1}{5.00} + \dfrac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$



$C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}}$

20.50 $U = \frac{1}{2}C(\Delta V)^2$

The circuit diagram is shown at the right.



(a) $C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$

$U = \frac{1}{2}\left(30.0 \times 10^{-6}\right)(100)^2 = \boxed{0.150\ \text{J}}$

(b) $C_s = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1} = \left(\dfrac{1}{25.0\ \mu\text{F}} + \dfrac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$U = \frac{1}{2}C(\Delta V)^2$

$\Delta V = \sqrt{\dfrac{2U}{C}} = \sqrt{\dfrac{2(0.150)}{4.17 \times 10^{-6}}} = \boxed{268\ \text{V}}$

20.54 (a) $C = \frac{\kappa \epsilon_0 A}{d} = \frac{2.10\left(8.85\times10^{-12}\text{ F/m}\right)\left(1.75\times10^{-4}\text{ m}^2\right)}{4.00\times10^{-5}\text{ m}} = 8.13\times10^{-11}\text{ F} = \boxed{81.3\text{ pF}}$

(b) $\Delta V_{\text{max}} = E_{\text{max}} d = \left(60.0\times10^{6}\text{ V/m}\right)\left(4.00\times10^{-5}\text{ m}\right) = \boxed{2.40\text{ kV}}$

***20.64** The original kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2\times10^{-16}\text{ kg}\right)\left(2\times10^{6}\text{ m/s}\right)^2 = 4.00\times10^{-4}\text{ J}$$

The potential difference across the capacitor is $\Delta V = \frac{Q}{C} = \frac{1000\ \mu\text{C}}{10\ \mu\text{F}} = 100\text{ V}$

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$U = q\Delta V = \left(-3\times10^{6}\text{ C}\right)\left(-100\text{ V}\right) = 3.00\times10^{-4}\text{ J}$$

Since its original kinetic energy is greater than this, $\boxed{\text{the particle will reach the negative plate}}$.

As the particle moves, the system keeps constant total energy

$(K+U)_{\text{at +plate}} = (K+U)_{\text{at −plate}}$: $\quad 4.00\times10^{-4}\text{ J} + \left(-3\times10^{-6}\text{ C}\right)\left(+100\text{ V}\right) = \frac{1}{2}\left(2\times10^{-16}\right)v_f^{\,2} + 0$

$$v_f = \sqrt{\frac{2\left(1.00\times10^{-4}\text{ J}\right)}{2\times10^{-16}\text{ kg}}} = \boxed{1.00\times10^{6}\text{ m/s}}$$

20.65 (a) We use Equation 20.29 to find the potential energy of the capacitor. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are

$$U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right) \qquad \text{and} \qquad U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$$

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$W = U_f - U_i = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$$

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i(\Delta V_i)$, and evaluate:

$$W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}\left(2.00\times10^{-9}\text{ F}\right)(100\text{ V})^2(5.00 - 1.00) = \boxed{4.00\times10^{-5}\text{ J}}$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$.

Substituting $C_f = \frac{C_i}{\kappa}$ and $Q = C_i(\Delta V_i)$ gives $\Delta V_f = \kappa \Delta V_i = 5.00(100\text{ V}) = \boxed{500\text{ V}}$

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

***20.71** Let C = the capacitance of an individual capacitor, and C_s represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\Delta V_{\text{charge}} = \left(500\times10^{-4}\text{ F}\right)(800\text{ V}) = 0.400\text{ C}$$

While being discharged in series,

$$\Delta V_{\text{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\text{ C}}{5.00\times10^{-5}\text{ F}} = \boxed{8.00\text{ kV}} \text{ (or 10 times the original voltage)}$$

21.1 $I = \frac{\Delta Q}{\Delta t}$ $\qquad$ $\Delta Q = I\Delta t = \left(30.0\times10^{-6}\text{ A}\right)(40.0\text{ s}) = 1.20\times10^{-3}\text{ C}$

$$N = \frac{Q}{e} = \frac{1.20\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = \boxed{7.50\times10^{15}\text{ electrons}}$$

21.5 $Q(t) = \int_0^t I\,dt = I_0\tau\left(1 - e^{-t/\tau}\right)$

(a) $Q(\tau) = I_0\tau\left(1 - e^{-1}\right) = \boxed{(0.632)I_0\tau}$

(b) $Q(10\tau) = I_0\tau\left(1 - e^{-10}\right) = \boxed{(0.99995)I_0\tau}$

(c) $Q(\infty) = I_0\tau\left(1 - e^{-\infty}\right) = \boxed{I_0\tau}$

21.6 We use $I = nqAv_d$ where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02\times10^{23}} = 4.49\times10^{-23}\text{ g/atom}$$

Thus, $$n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\text{ g/cm}^3}{4.49\times10^{-23}\text{ g/atom}}$$

$$n = 6.02\times10^{22}\text{ atoms/cm}^3 = 6.02\times10^{28}\text{ atoms/m}^3$$

Therefore, $$v_d = \frac{I}{nqA} = \frac{5.00\text{ A}}{\left(6.02\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{-6}\text{ m}^2\right)} = 1.30\times10^{-4}\text{ m/s}$$

or, $v_d = \boxed{0.130\text{ mm/s}}$

21.8 (a) Given $M = \rho_d V = \rho_d A\ell$ where $\rho_d \equiv$ mass density,

we obtain: $A = \dfrac{M}{\rho_d \ell}$ Taking $\rho_r \equiv$ resistivity, $R = \dfrac{\rho_r \ell}{A} = \dfrac{\rho_r \ell}{M/\rho_d \ell} = \dfrac{\rho_r \rho_d \ell^2}{M}$

Thus, $$\ell = \sqrt{\frac{MR}{\rho_r \rho_d}} = \sqrt{\frac{\left(1.00\times10^{-3}\right)(0.500)}{(1.70\times10^{-8})(8.92\times10^3)}}$$ $\ell = \boxed{1.82\text{ m}}$

(b) $V = \dfrac{M}{\rho_d}$, or $\pi r^2 \ell = \dfrac{M}{\rho_d}$

Thus, $$r = \sqrt{\frac{M}{\pi \rho_d \ell}} = \sqrt{\frac{1.00\times10^{-3}}{\pi(8.92\times10^3)(1.82)}}$$ $r = 1.40\times10^{-4}\text{ m}$

The diameter is twice this distance: diameter $= \boxed{280\ \mu\text{m}}$

***21.11** (a) $\rho = \rho_0\left[1+\alpha(T-T_0)\right] = \left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+3.90\times10^{-3}(30.0°)\right] = \boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times10^{6}\ \text{A/m}^2}$

(c) $I = JA = J\left(\dfrac{\pi d^2}{4}\right) = \left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\dfrac{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2}{4}\right] = \boxed{49.9\ \text{mA}}$

(d) $n = \dfrac{6.02\times10^{23}\ \text{electrons}}{\left(\dfrac{26.98\ \text{g}}{2.70\times10^{6}\ \text{g}/\text{m}^3}\right)} = 6.02\times10^{28}\ \text{electrons}/\text{m}^3$

$v_d = \dfrac{J}{ne} = \dfrac{(6.35\times10^{6}\ \text{A/m}^2)}{(6.02\times10^{28}\ \text{electrons/m}^3)(1.60\times10^{-19}\ \text{C})} = \boxed{659\ \mu\text{m/s}}$

(e) $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$

21.13 $\rho = \dfrac{m}{nq^2\tau}$

so $\tau = \dfrac{m}{\rho n q^2} = \dfrac{\left(9.11\times10^{-31}\right)}{\left(1.70\times10^{-8}\right)\left(8.49\times10^{28}\right)\left(1.60\times10^{-19}\right)^2} = 2.47\times10^{-14}\ \text{s}$

$v_d = \dfrac{qE}{m}\tau$

so $7.84\times10^{-4} = \dfrac{\left(1.60\times10^{-19}\right)E\left(2.47\times10^{-14}\right)}{9.11\times10^{-31}}$

Therefore, $E = \boxed{0.181\ \text{V/m}}$

***21.22** You pay the electric company for energy transferred in the amount $E = \mathcal{P}\Delta t$.

(a) $$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{86400\text{ s}}{1\text{ d}}\right)\left(\frac{1\text{ J}}{1\text{ W}\cdot\text{s}}\right) = 48.4\text{ MJ}$$

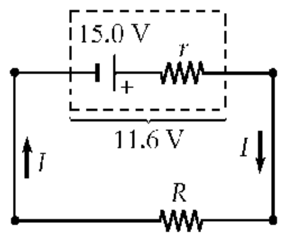
$$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1000}\right) = 13.4\text{ kWh}$$

$$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{0.12\text{ \$}}{\text{kWh}}\right) = \boxed{\$\ 1.61}$$

(b) $$\mathcal{P}\Delta t = 970\text{ W}(3\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{0.12\text{ \$}}{\text{kWh}}\right) = \boxed{\$\ 0.00582} = 0.582\ \text{¢}$$

(c) $$\mathcal{P}\Delta t = 5200\text{ W}(40\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{0.12\text{ \$}}{\text{kWh}}\right) = \boxed{\$\ 0.416}$$

21.23 (a) $$\mathcal{P} = \frac{(\Delta V)^2}{R}$$



becomes $$20.0\text{ W} = \frac{(11.6\text{ V})^2}{R}$$

so $$R = \boxed{6.73\ \Omega}$$

(b) $$\Delta V = IR$$

so $$11.6\text{ V} = I(6.73\ \Omega)$$

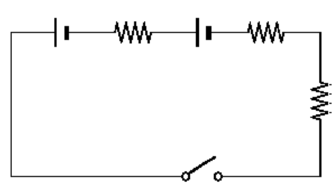
and $$I = 1.72\text{ A}$$

$$\mathcal{E} = IR + Ir$$

so $$15.0\text{ V} = 11.6\text{ V} + (1.72\text{ A})r$$

$$r = \boxed{1.97\ \Omega}$$

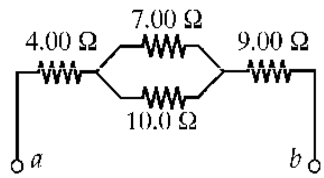
21.24 The total resistance is $R = \dfrac{3.00\text{ V}}{0.600\text{ A}} = 5.00\ \Omega$



(a) $$R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$$

(b) $$\frac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \frac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$$

21.25 (a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$



$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for $4.00\ \Omega$, $9.00\ \Omega$ resistors

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

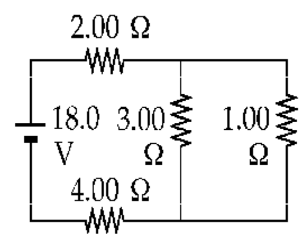
$8.18\ \text{V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\ \text{A}}$ for $7.00\ \Omega$ resistor

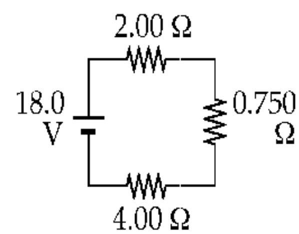
$8.18\ \text{V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\ \text{A}}$ for $10.0\ \Omega$ resistor

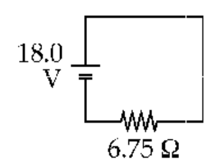
***21.29** $R_p = \left(\dfrac{1}{3.00} + \dfrac{1}{1.00}\right)^{-1} = 0.750\ \Omega$



$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$



$I_{\text{battery}} = \dfrac{\Delta V}{R_s} = \dfrac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$



$\mathcal{P} = I^2R$: $\mathcal{P}_2 = (2.67\ \text{A})^2(2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\ \text{W}}$ in $2.00\ \Omega$

$\mathcal{P}_4 = (2.67\ \text{A})^2(4.00\ \text{A}) = \boxed{28.4\ \text{W}}$ in $4.00\ \Omega$

$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V},$

$\Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$

$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V}\ (= \Delta V_3 = \Delta V_1)$

$\mathcal{P}_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}$ in $3.00\ \Omega$

$\mathcal{P}_1 = \dfrac{(\Delta V_1)}{R_1} = \dfrac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}$ in $1.00\ \Omega$

***21.47** (a) $\mathbf{E} = -\frac{dV}{dx}\mathbf{i} = -\frac{(0-4.00)\text{ V}}{(0.500-0)\text{ m}} = \boxed{8.00\mathbf{i}\text{ V/m}}$

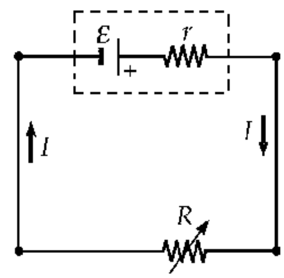
(b) $R = \frac{\rho \ell}{A} = \frac{\left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)(0.500\text{ m})}{\pi\left(1.00\times10^{-4}\text{ m}\right)^2} = \boxed{0.637\ \Omega}$

(c) $I = \frac{\Delta V}{R} = \frac{4.00\text{ V}}{0.637\ \Omega} = \boxed{6.28\text{ A}}$

(d) $\boldsymbol{J} = \frac{I}{A}\mathbf{i} = \frac{6.28\text{ A}}{\pi\left(1.00\times10^{-4}\text{ m}\right)^2} = 2.00\times10^8\,\mathbf{i}\text{ A/m}^2 = \boxed{200\mathbf{i}\text{ MA/m}^2}$

(e) $\rho\boldsymbol{J} = \left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\times10^8\,\mathbf{i}\text{ A/m}^2\right) = 8.00\mathbf{i}\text{ V/m} = \mathbf{E}$

***21.54** The current in the simple loop circuit will be $I = \frac{\mathcal{E}}{R+r}$



(a) $\Delta V_{ter} = \mathcal{E} - Ir = \frac{\mathcal{E}R}{R+r}$ and $\Delta V_{ter} \to \mathcal{E}$ as $\boxed{R\to\infty}$

(b) $I = \frac{\mathcal{E}}{R+r}$ and $I \to \frac{\mathcal{E}}{r}$ as $\boxed{R\to 0}$

(c) $\mathcal{P} = I^2R = \mathcal{E}^2\frac{R}{(R+r)^2}$ $\frac{d\mathcal{P}}{dR} = \frac{-2\mathcal{E}^2R}{(R+r)^3} + \frac{\mathcal{E}^2}{(R+r)^2} = 0$

Then $2R = R + r$ and $\boxed{R = r}$