$$+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$$

$$5.00 = 7.00 I_1$$

$$I_1 = 0.714 \text{ A}$$

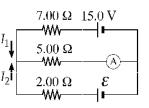
$$I_3 = I_1 + I_2 = 2.00 \text{ A}$$

$$0.714 + I_2 = 2.00$$

$$I_2 = 1.29 \text{ A}$$

$$+\mathcal{E} - 2.00(1.29) - 5.00(2.00) = 0$$

$$\varepsilon$$
 = 12.6 V



 $3.00 \Omega$ 

## 21.33 We name currents $I_1$ , $I_2$ , and $I_3$ as shown.

From Kirchhoff's current rule,  $I_3 = I_1 + I_2$ 

Applying Kirchhoff's voltage rule to the loop containing  $I_2$  and  $I_3$ ,

12.0 V 
$$-(4.00)I_3 - (6.00)I_2 - 4.00$$
 V = 0

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing  $I_1$  and  $I_2$ ,

$$-(6.00)I_2 - 4.00 \text{ V} + (8.00)I_1 = 0$$

$$(8.00)I_1 = 4.00 + (6.00)I_2$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases}
8 = 4I_1 + 4I_2 + 6I_2 \\
8I_1 = 4 + 6I_2
\end{cases}$$

$$\begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation  $8 = 4I_1 + 13.3I_1 - 6.67$ 

$$I_1 = \frac{14.7 \text{ V}}{17.3 \Omega} = 0.846 \text{ A}$$

Then 
$$I_2 = 1.33(0.846 \text{ A}) - 0.667$$

and

$$I_3 = I_1 + I_2$$

$$I_1$$
 = 846 mA,  $I_2$  = 462 mA,  $I_3$  = 1.31 A

All currents are in the directions indicated by the arrows in the circuit diagram.

(a) By the 4.00-V battery:

$$\Delta U = (\Delta V)I \Delta t = (4.00 \text{ V})(-0.462 \text{ A})120 \text{ s} = \boxed{-222 \text{ J}}$$

By the 12.0-V battery:

$$(12.0 \text{ V})(1.31 \text{ A})120 \text{ s} = \boxed{1.88 \text{ kJ}}$$

(b) By the 8.00- $\Omega$  resistor:

$$I^2R\Delta t = (0.846 \text{ A})^2(8.00 \Omega)120 \text{ s} = 687 \text{ J}$$

By the 5.00-  $\Omega$  resistor:

$$(0.462 \text{ A})^2 (5.00 \Omega) 120 \text{ s} = \boxed{128 \text{ J}}$$

By the 1.00-  $\Omega$  resistor:

$$(0.462 \text{ A})^2 (1.00 \Omega) 120 \text{ s} = 25.6 \text{ J}$$

By the 3.00-  $\Omega$  resistor:

$$(1.31 \text{ A})^2 (3.00 \Omega) 120 \text{ s} = 616 \text{ J}$$

By the 1.00-  $\Omega$  resistor:

$$(1.31 \text{ A})^2 (1.00 \Omega) 120 \text{ s} = 205 \text{ J}$$

(c) -222 J + 1.88 kJ = 1.66 kJ from chemical to electrical.

687 J + 128 J + 25.6 J + 616 J + 205 J = 1.66 kJ from electrical to internal.

Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and

$$(1.71R)I_1 + (3.71R)I_2 = 500$$

With  $R = 1000 \Omega$ , simultaneous solution of these equations yields:

$$I_1 = 10.0 \text{ mA}$$

and

$$I_2 = 130.0 \text{ mA}$$

From Figure (b),

$$V_c - V_a = (I_1 + I_2)(1.71R) = 240 \text{ V}$$

Thus, from Figure (a),

$$I_4 = \frac{V_c - V_a}{4R} = \frac{240 \text{ V}}{4000 \Omega} = 60.0 \text{ mA}$$

Finally, applying Kirchhoff's point rule at point *a* in Figure (a) gives:

$$I = I_4 - I_1 = 60.0 \text{ mA} - 10.0 \text{ mA} = +50.0 \text{ mA},$$

or

$$I = 50.0 \text{ mA from point } a \text{ to point } e$$
.

**21.38** (a)  $I(t) = -I_0 e^{-t/RC}$ 

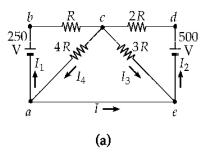
$$I_0 = \frac{Q}{RC} = \frac{5.10 \times 10^{-6} \text{ C}}{(1300 \Omega)(2.00 \times 10^{-9} \text{ F})} = 1.96 \text{ A}$$

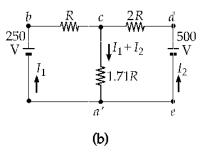
$$I(t) = -(1.96 \text{ A}) \exp \left[ \frac{-9.00 \times 10^{-6} \text{ s}}{(1300 \Omega)(2.00 \times 10^{-9} \text{ F})} \right] = \boxed{-61.6 \text{ mA}}$$

(b) 
$$q(t) = Qe^{-t/RC} = (5.10 \ \mu\text{C}) \exp\left[\frac{-8.00 \times 10^{-6} \text{ s}}{(1300 \ \Omega)(2.00 \times 10^{-9} \text{ F})}\right] = \boxed{0.235 \ \mu\text{C}}$$

(c) The magnitude of the maximum current is

$$I_0 = 1.96 \text{ A}$$





**21.39** (a) 
$$RC = (1.00 \times 10^6 \ \Omega)(5.00 \times 10^{-6} \ F) = \boxed{5.00 \ s}$$

(b) 
$$Q = C\mathcal{E} = (5.00 \times 10^{-6} \text{ C})(30.0 \text{ V}) = \boxed{150 \ \mu\text{C}}$$

$$C = \frac{1}{5.00 \, \mu \text{F}}$$

(c) 
$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} = \left(\frac{30.0}{1.00 \times 10^6}\right) \exp\left[\frac{-10.0}{\left(1.00 \times 10^6\right)\left(5.00 \times 10^{-6}\right)}\right] = \boxed{4.06 \ \mu\text{A}}$$

\*21.46 (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R<sub>2</sub> we have

$$\mathcal{P} = I^2 R_2$$
  $I = \sqrt{\frac{\mathcal{P}}{R_2}} = \sqrt{\frac{2.40 \text{ V} \cdot \text{A}}{7000 \text{ V/A}}} = 18.5 \text{ mA}$ 

The potential difference across  $R_1$  and  $C_1$  is

$$\Delta V = IR_1 = (1.85 \times 10^{-2} \text{ A})(4000 \text{ V/A}) = 74.1 \text{ V}$$

The charge on  $C_1$  is

$$Q = C_1 \Delta V = (3.00 \times 10^{-6} \text{ C/V})(74.1 \text{ V}) = \boxed{222 \ \mu\text{C}}$$

The potential difference across  $R_2$  and  $C_2$  is

$$\Delta V = IR_2 = (1.85 \times 10^{-2} \text{ A})(7000 \Omega) = 130 \text{ V}$$

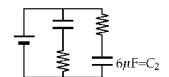
The charge on  $C_2$  is

$$Q = C_2 \Delta V = (6.00 \times 10^{-6} \text{ C/V})(130 \text{ V}) = 778 \ \mu\text{C}$$

The battery emf is

$$IR_{eq} = I(R_1 + R_2) = 1.85 \times 10^{-2} \text{ A}(4000 + 7000) \text{ V/A} = 204 \text{ V}$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge  $\mathcal{C}_2$  is



$$Q = C_2 \Delta V = (6.00 \times 10^{-6} \text{ C/V})(204 \text{ V}) = 1222 \ \mu\text{C}$$

for a change of 1222 
$$\mu$$
C – 778  $\mu$ C = 444  $\mu$ C

\*21.56 (a) 
$$q = C\Delta V (1 - e^{-t/RC})$$

$$q = (1.00 \times 10^{-6} \text{ F})(10.0 \text{ V}) \left[ 1 - e^{\frac{-10.0}{(2.00 \times 10^{-6})(1.00 \times 10^{-6})}} \right] = \boxed{9.93 \ \mu\text{C}}$$

(b) 
$$I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$$

$$I = \left(\frac{10.0 \text{ V}}{2.00 \times 10^6 \Omega}\right) e^{-5.00} = 3.37 \times 10^{-8} \text{ A} = \boxed{33.7 \text{ nA}}$$

(c) 
$$\frac{dU}{dt} = \frac{d}{dt} \left( \frac{1}{2} \frac{q^2}{C} \right) = \left( \frac{q}{C} \right) \frac{dq}{dt} = \left( \frac{q}{C} \right) I$$

$$\frac{dU}{dt} = \left(\frac{9.93 \times 10^{-6} \text{ C}}{1.00 \times 10^{-6} \text{ C/V}}\right) \left(3.37 \times 10^{-8} \text{ A}\right) = 3.34 \times 10^{-7} \text{ W} = \boxed{334 \text{ nW}}$$

(d) 
$$\mathcal{P}_{\text{battery}} = I\mathcal{E} = (3.37 \times 10^{-8} \text{ A})(10.0 \text{ V}) = 3.37 \times 10^{-7} \text{ W} = 337 \text{ nW}$$