AC Circuit Homework.

1) (a) $V(t) = I(t)\cdot R \Rightarrow V_{RMS} = I_{RMS}\cdot R$. Also we know

$P = I_{RMS}^2 R = I_{RMS}\cdot V_{RMS}$

$I_{RMS} = P/V_{RMS} = 60W/115V$ $\boxed{I_{RMS} = 0.52\,A}$

(b) $I_0 = \sqrt{2}\, I_{RMS} \Rightarrow \boxed{I_0 = 0.74\,A}$

(c) $P_{max} = I_0^2 R = (\sqrt{2}\, I_{RMS})^2 R = 2 I_{RMS}^2 R \Rightarrow \boxed{P_{max} = 120\,W}$

2) (a) $\omega = 2\pi f = 2\pi\cdot(60/s) \Rightarrow \boxed{\omega = 377/s}$

(b) $I_0 = V_0/R = 20V/3\Omega \Rightarrow \boxed{I_0 = 6.67\,A}$

$I_{rms} = I_0/\sqrt{2}$ $\boxed{I_{rms} = 4.71\,A}$

(c) $P = (I_{rms})^2\cdot R = (4.71A)^2\cdot 3\Omega$ $\boxed{P = 66.7\,W}$

3) (a) $X_L = \omega L$

$f = 60/s \Rightarrow \omega = 377/s$ $X_L = (377/s)(10^{-3}H) = \boxed{0.377\,\Omega}$

$f = 6\times10^3/s$ $\omega = 3.77\times10^4/s$ $\boxed{X_L = 37.7\,\Omega}$

(b) $X_C = 1/\omega C$

$f = 60/s \Rightarrow X_C = [(377/s)\cdot(10^{-5}F)]^{-1} \Rightarrow \boxed{X_C = 265\,\Omega}$

$f = 6\times10^3/s \Rightarrow \boxed{X_C = 2.65\,\Omega}$

4) $X_L = \omega L = (2\pi)\cdot(60/s)(0.050) = 18.85\,\Omega$

For an LRC circuit, the impedance is $[R^2 + (X_L - X_C)^2]^{\frac{1}{2}}$
and so if there is no capacitor we have

$Z = [R^2 + X_L^2]^{1/2} = [(10\Omega)^2 + (18.85\Omega)^2]^{1/2} = 21.3\,\Omega$

$I_{RMS} = V_{RMS}/Z = (115V)/(21.3\Omega)$ $\boxed{I_{RMS} = 5.4\,A}$

5) The frequency we want is $f = 1050\,kHz = 1.05 \times 10^6\ Hz$

$\Rightarrow \quad \omega = 2\pi f = 6.6 \times 10^6/s$

The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$\Rightarrow$

$$LC = \frac{1}{\omega^2} = 2.3 \times 10^{-14}\, s^2$$

If $L = 10^{-3}H$ then $C = 2.3\times10^{-14} s^2/10^{-3} = 2.3 \times 10^{-11} F$

$\boxed{C = 23\,pF}$ $\quad p = pico = 10^{-12}$

6) (a) $U = \frac{1}{2}CV^2 = \frac{1}{2}(5\times10^{-6}F)(30V)^2 \Rightarrow$ $\boxed{U = 2.25\times10^{-3} J}$

(b) The natural oscillation frequency is

$$\omega = \frac{1}{\sqrt{LC}} = \left[(10^{-2}H)(5\times10^{-6}F)\right]^{-\frac{1}{2}} = 4472/s$$

$\boxed{\omega = 4472/s}$ OR $\boxed{f = \omega/2\pi = 712\,Hz}$

(c) The energy is transferred back and forth between the capacitor and the inductor. The peak current occurs when $U_C = 0$ then

$$U_L = \frac{1}{2}LI^2 = 2.25\times10^{-3} J$$

$$I = \left[(2)(2.25\times10^{-3}J)/(0.01\,H)\right]^{\frac{1}{2}}$$ $\boxed{I_{max} = 0.67\,A}$

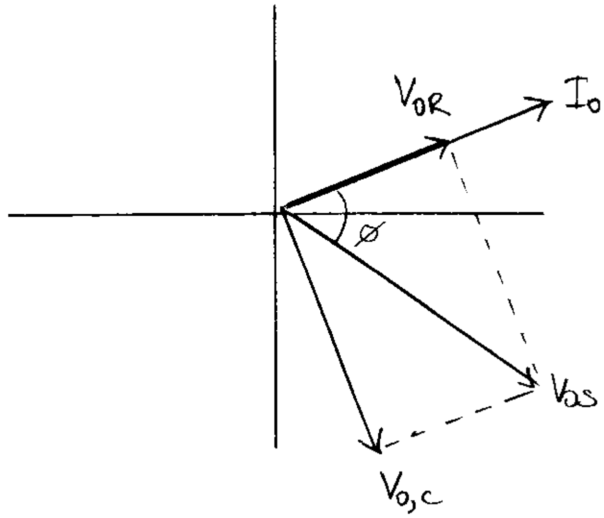
7) (a) We need to remember that for a capacitor, the voltage trails the current by $90°$. We will also use

$$X_C = 1/\omega C = 1/(3000/s)(5\times10^{-6}F) = 66.7\,\Omega$$

This tells us that

$$V_{0,C} = X_C I_0 = 66.7\,\Omega \cdot I_0 \qquad V_{0R} = I_0 \cdot R = 40\,\Omega \cdot I_0$$

The phasor diagram is shown at the right



(b) $V_{0,S} = \left[V_{0R}^2 + V_{0,C}^2\right]^{\frac{1}{2}}$

$= \left[(RI_0)^2 + (X_C I_0)^2\right]^{\frac{1}{2}}$

$= I_0 \left[R^2 + X_C^2\right]^{\frac{1}{2}}$

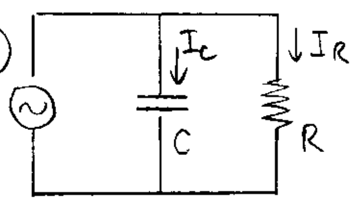
So

$I_0 = (5V) / \left[(40\Omega)^2 + (66.7\Omega)^2\right]^{\frac{1}{2}}$ $\boxed{I_0 = 64\,mA}$

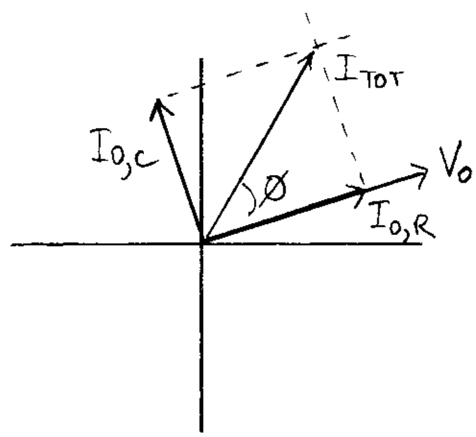
(c) From the picture the <u>current leads the voltage</u> by an angle ϕ where

$\tan\phi = V_{0C}/V_{0R} = X_C/R = 66.7\Omega/40\Omega$ $\boxed{\phi = 59°}$

8) (a)



Notice that the 5V AC is connected directly across both the capacitor and the resistor, so we have



$I_{0,c} = 5V/X_C = 0.075A$

$I_{0,R} = 5V/R = 0.125\,A$

(b) From the drawing

$I_0 = \left[(I_{0,c})^2 + (I_{0,R})^2\right]^{\frac{1}{2}} = \left[(75mA)^2 + (125mA)^2\right]^{\frac{1}{2}}$

$\boxed{I_0 = 146\,mA}$

(c) Again the <u>current leads the voltage</u>, and the phase angle is ϕ where

$\tan\phi = I_{0C}/I_{0R} = 75/125 = 0.6$ $\boxed{\phi = 31°}$

9) (a) The resonant frequency is $\omega = \frac{1}{\sqrt{LC}}$

$$\omega = \left[(3\times10^{-9}\,F)\cdot(5\times10^{-5}\,H)\right]^{-\frac{1}{2}} = \underline{2.582\times10^{6}/s}$$

At the resonant frequency

$$X_C = X_L = \omega L = (2.582\times10^{6}/s)(5\times10^{-5}\,H) = \underline{129\,\Omega}$$

$$Z = \left[R^2 + (X_L - X_C)^2\right]^{1/2} = R = 5\Omega$$

so

$$I_0 = 5V/5\Omega \qquad \boxed{I_0 = 1A}$$

(b) Here $X_L = \omega L = (0.8)\,\omega_0 L = (0.8)(129\,\Omega) = 103.2\,\Omega$

$$X_C = 1/\omega C = 1/0.8\,\omega_0 C = \left(\tfrac{1}{0.8}\right)(129\,\Omega) = 161.2$$

$$Z = \left[R^2 + (X_L - X_C)^2\right]^{\frac{1}{2}} = \left[(5\Omega)^2 + (103\Omega - 161\Omega)^2\right]^{\frac{1}{2}} = 58\,\Omega$$

$\Rightarrow$

$$I_0 = 5V/58\Omega \qquad \boxed{I_0 = 0.086\,A}$$

(c) At $1.2\cdot\omega_0$

$$X_L = (1.2)\cdot(129\,\Omega) = 154.8\,\Omega$$

$$X_C = (129\,\Omega)/1.2 = 107.5$$

$$Z = 47.6\,\Omega \qquad \boxed{I_0 = 0.105\,A}$$