EXAM 1 REVIEW

Chapter 1:

- Newton's Laws:
  1) In the absense of forces, particles move at constant $\vec{v}$
  2) $\vec{F} = m\vec{a}$
  3) $\vec{F}_{12} = -\vec{F}_{21}$
- An inertial frame is any frame of reference in which Law 1 holds.
- Momentum Conservation: For any system of particles $\frac{d}{dt}\vec{P}_{\text{tot}} = \vec{F}_{\text{ext}}$
- General solution method for time dependent forces, $F = F(t)$ – – write $m\frac{dv}{dt} = F(t)$, rearrange to separate variables, and integrate:

  $$v(t) = v_0 + \frac{1}{m}\int_0^t F(t')dt'$$

  To find $x(t)$, integrate one more time.
- Polar coordinates:

  $$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\hat{r} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi}$$

Chapter 2:

- Drag Forces: $\vec{F} = -f(v)\hat{v}$, where $f(v) = bv + cv^2$
- Projectile motion with linear drag:
  Horizontal motion: $m\dot{v}_x = -bv_x$ – – linear, first order, homogenous differential equation – easily solved for $v(t)$ with an exponential function
  Vertical motion: $m\dot{v}_y = mg - bv_y$ – – same equation except inhomogenous – find particular solution and combine with general solution of homogenous equation
- General solution method for velocity dependent forces – write $m\frac{dv}{dt} = F(v)$, rearrange to separate variables, and integrate:

  $$t = m\int_{v_0}^{v} \frac{dv}{F(v)}$$

  Solve for $v(t)$ by algebra. Then integrate one more time to find $x(t)$.
- Problems with $F = F(x)$ can be solved for $v(x)$ by writing $m\frac{dv}{dt} = mv\frac{dv}{dx}$. Separate variables and integrate:

  $$m\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} F(x)dx$$

  This works for velocity dependent forces as well, with $F(v)$ in the v integral.
- Motion in a uniform magnetic field:

  $$v_x = v_{x0}\cos\omega t + v_{y0}\sin\omega t \qquad v_y = v_{y0}\cos\omega t - v_{x0}\sin\omega t$$

  where $\omega = qB/m$. This is circular motion with $R = mv/qB$.
- Motion in crossed electric/magnetic field: Circular motion (as above) superimposed on uniform drift $\perp$ to both $\vec{E}$ and $\vec{B}$.

Chapter 3:

- Rocket equation: $m\dfrac{dv}{dt} = -v_{\text{ex}}\dfrac{dm}{dt} + F_{\text{ext}}$
- Center of Mass: $\vec{R} = \dfrac{1}{M}\sum_{\alpha} m_\alpha \vec{r}_\alpha.$ This leads to $\vec{P}_{\text{tot}} = M\dfrac{d}{dt}\vec{R}$ and $\vec{F}_{\text{ext}} = M\dfrac{d^2}{dt^2}\vec{R}.$
- Angular Momentum: Single particle: $\vec{\Gamma} = \dfrac{d}{dt}\vec{\ell}$ For a system of particles: $\vec{\Gamma}_{\text{ext}} = \dfrac{d}{dt}\vec{L}$
- Rotation about an axis: $L = I\omega$ where $I = \sum_{\alpha} m_\alpha \rho_\alpha^2$
- Parallel axis theorem: $I = I_{\text{cm}} + M\rho^2$
- Two-body collisions – apply energy and momentum conservation
- CM Frame of reference – moves along with CM of the system $\vec{v}_\alpha = \vec{v}'_\alpha + \vec{v}_{\text{cm}}$

  Total momentum in the cm frame is zero.

Chapter 4:

- Work energy theorem: Gain in kinetic energy = work done on m – – $W = \int \vec{F} \cdot d\vec{s}$
- Potential energy: $U(\vec{r}) = -\int_{\vec{r}_s}^{\vec{r}} \vec{F}(\vec{r}) \cdot d\vec{s}$ $\vec{F} = -\vec{\nabla} U$
- Energy $(T+U)$ is conserved (and definition of U makes sense) if $\vec{\nabla} \times \vec{F} = 0$
- Problem solving:

  $T + U = E$ (constant determined from initial conditions) gives v as a function of position.

  Constraint forces have no effect on E

  $\dfrac{d}{dt}[T+U] = 0$ may give a useful equation of motion
- Central forces are always conservative: $U(r) = -\int_{r_s}^{r} F(r)dr$
- Two-body system with no external forces and $\vec{F}_{12} = -\vec{F}_{21} = F(r)\hat{r}$ (where $\vec{r} = \vec{r}_1 - \vec{r}_2$)

  $E = T_1 + T_2 + U(r)$ is conserved

  Internal motion given by $\vec{F} = \mu\dfrac{d^2}{dt^2}\vec{r}$ where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$