DISCUSSION 4/15/04

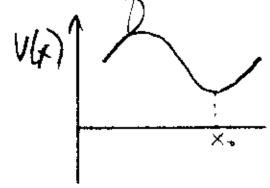
Review for Exam

CHAPTER 5 $\Rightarrow$ Oscillations

Beginning with SHM $\quad F = m\ddot{x} = -kx$

$$\Rightarrow \quad x(t) = A\cos(\omega t - \phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

$\Rightarrow$ Good approx for oscillations about any stable equilibrium pt.
if amplitude is small



- Next add a linear damping force

$$F = -bv \quad \Rightarrow$$

$$m\ddot{x} + b\dot{x} + kx = 0$$

OR

$$D\,x(t) = 0 \qquad \text{where} \quad D = m\frac{d^2}{dt^2} + b\frac{d}{dt} + m$$

Solutions:

Strong damping: $\beta = \frac{b}{2m} > \omega_0 \qquad x(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$

$$\alpha = \beta \pm [\beta^2 - \omega_0^2]^{\frac{1}{2}}$$

Weak damping $\quad \beta < \omega_0 \qquad x(t) = A e^{-\beta t}\cos(\omega_1 t - \phi)$

$$\omega_1 = [\omega_0^2 - \beta^2]^{\frac{1}{2}}$$

Critical $\quad \beta = \omega_0 \qquad x(t) = Ae^{-\beta t} + Bte^{-\beta t}$

- Forced H.O. $\qquad m\ddot{x} + b\dot{x} + kx = F(t)$

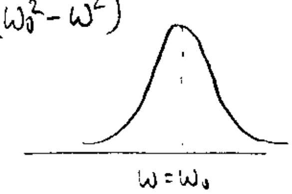
$F(t) = f_0 \cos\omega t$

$$\Rightarrow \qquad x(t) = \underbrace{B\cos(\omega_1 t - \phi)e^{-\beta t}}_{\text{transient}} + A\cos(\omega t - \delta)$$

where

$$A = \frac{f_0}{[(\omega_0^2 - \omega^2) + (2\beta\omega)^2]^{1/2}} \qquad \tan\delta = \frac{2\beta\omega}{(\omega_0^2 - \omega^2)}$$

$\hookrightarrow$ resonance behavior

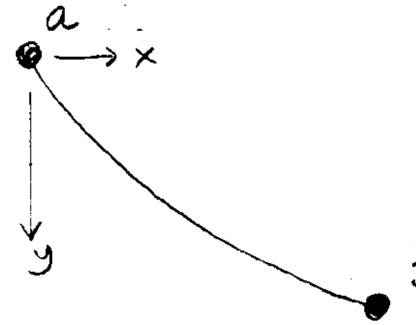


Arbitrary F(t)

$$x(t) = \left(\frac{1}{m\omega_1}\right)\int_{-\infty}^{t} F(t')\sin\omega_1(t-t')e^{-\beta(t-t')}\,dt'$$

CHAPTER 6: Calculus of Variations

Typical kind of problem - Find the path (thru space) that minimizes time for rollercoaster to travel from a to b



$$dt = \frac{ds}{v}$$

$$\frac{1}{2}mv^2 = mgy \Rightarrow v = \sqrt{2gy}$$

$$t = \int \frac{1}{\sqrt{2gy}}\left[dx^2 + dy^2\right]^{\frac{1}{2}} = \text{minimum}$$

$\Rightarrow$ means nearby paths have nearly the same t

$$\boxed{\delta \int \frac{1}{\sqrt{2gy}}\left[dx^2 + dy^2\right]^{\frac{1}{2}} = 0}$$

e.g. use x as indep. variable: $y = y(x)$

$$\delta \int_{x_1}^{x_2} \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}}{[2gy]^{1/2}}\, dx = 0$$

$$\delta \int_{x_1}^{x_2} F(y, y'; x)\, dx = 0$$

$$\boxed{\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0}$$

CHAP 7 — HAMILTONS PRINCIPLE:

$$\delta \int \mathcal{L}\, dt = \delta \int \mathcal{L}(q, \dot{q}, t)\, dt = 0$$

$\Rightarrow$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0$$

gives equations of motion

Multiple coords $\Rightarrow$ same eq for each.

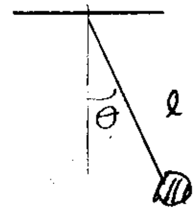
$$\boxed{\mathcal{L} = T - U}$$

OK - good technique to find equations of motion in many complex systems

Definitions: $p_k = \frac{\partial \mathcal{L}}{\partial \dot{q}_k}$ = generalized momentum

$Q_k = \frac{\partial \mathcal{L}}{\partial q_k}$ = generalized force $\qquad \dot{p}_k = Q_k$

$v = l\dot{\theta} \Rightarrow \quad T = \frac{1}{2} m l^2 \dot{\theta}^2$



$$U = mgl(1-\cos\theta)$$

$$\mathcal{L} = \frac{1}{2} m l^2 \dot{\theta}^2 - mgl + mgl\cos\theta$$

$p_\theta = ml^2\dot{\theta}$ $\qquad Q_\theta = -mgl\sin\theta$ $\qquad ml^2\ddot{\theta} = -mgl\sin\theta$

angular mom. $\qquad$ torque

CONSTANTS + CONSERVATION LAWS.

1) If $\partial\mathcal{L}/\partial q_k = 0$ then $\frac{d}{dt} p_k = 0 \Rightarrow p_k = \text{constant}$

2) (a) Isolated system $\Rightarrow$ all locations in space are equivalent
$\Rightarrow \delta\mathcal{L} = 0$ if $\vec{r}_\alpha \to \vec{r}_\alpha + \vec{R}$ for all α
$\Rightarrow \vec{P}_{TOT} = \text{constant}$

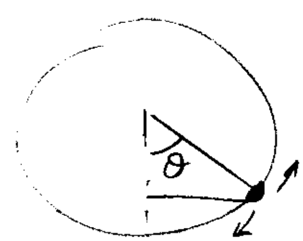
(b) All directions in space are equivalent $\qquad \vec{L}_{TOT} = \text{constant}$

3) If $\underbrace{\frac{\partial\mathcal{L}}{\partial t} = 0}_{\text{very often}}$ then $H \equiv \underbrace{\sum_k p_k \dot{q}_k - \mathcal{L}}_{\text{often } T+U} = \text{constant}$

- Good Example Problem:

Bead on a rotating hoop $\qquad T = \frac{1}{2}$

$$T = \frac{1}{2} m (R\dot{\theta})^2 + \frac{1}{2} m (R\sin\theta\,\omega)^2$$



$$U = mgR(1-\cos\theta)$$

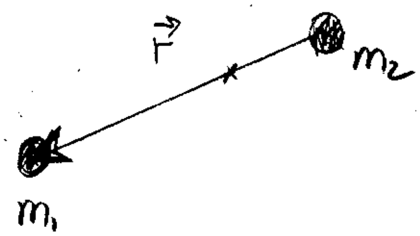
$$\mathcal{L} = \frac{1}{2} mR^2\dot{\theta}^2 + \frac{1}{2} m\omega^2 R^2 \sin^2\theta - mgR + mgR\cos\theta.$$

① Eq. of motion

② $H = \text{constant}$. $\qquad \frac{1}{2} m\dot{x}^2 + U(x) = E$ - same form.

$$H = \frac{1}{2} mR^2\dot{\theta}^2 - \frac{1}{2} m\omega^2 R^2 \sin^2\theta + mgR - mgR\cos\theta. = K.$$

CHAP 8 TWO BODY CENTRAL FORCE PROBLEM.



$$\mathcal{L} = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2}\mu \dot{r}^2 - U(r)$$

↳ C.M. moves with const. vel.

$\vec{L}$ conserved ⇒ motion confined to a plane

⇒

$$\mathcal{L} = \frac{1}{2}\mu\left[\dot{r}^2 + r^2\dot{\phi}^2\right] - U(r)$$

⇒

① $\underline{l = \mu r^2 \dot{\phi}}$ = constant

② $$\boxed{\mu \ddot{r} = \frac{l^2}{\mu r^3} - \frac{\partial U}{\partial r}}$$

③ $$\boxed{H = \frac{1}{2}\mu \dot{r}^2 + \underbrace{\frac{l^2}{2\mu r^2} + U(r)} = E\,(\text{constant})}$$

$$U_{eff} = U(r) + \frac{l^2}{2\mu r^2}$$

ORBIT EQUATION: $u \equiv \frac{1}{r}$

$$\boxed{\frac{d^2 u}{d\phi^2} = -u - \left(\frac{\mu}{l^2}\right)\frac{1}{u^2} F\left(\frac{1}{u}\right)}$$

PLANETS: $F = -\frac{K}{r^2}$ ⇒ $r = \frac{C}{1 + \epsilon \cos\phi}$

where

$$C = l^2/K\mu \quad , \quad E = \frac{K^2 \mu}{2 l^2}\left(\epsilon^2 - 1\right)$$

$\epsilon = 0$ ⇒ circle ; $0 < \epsilon < 1$ ⇒ ellipse ; $\epsilon = 1$ parabola
$\epsilon > 1$ ⇒ hyperbola

Kepler's Laws Follow from above.

CHAPTER 9 – NON-INERTIAL FRAMES

1) Accelerating frames $\boxed{m\ddot{\vec{r}} = \vec{F} - m\vec{A}}$

↳ "inertial force"

Constant acceleration ⇒ $\vec{F}_{inertial}$ equivalent to uniform gravitational field

- Tidal Forces

$$\vec{F}_{tidal} = G M_{moon} m \left[\frac{\hat{d}_0}{d_0^2} - \frac{\hat{d}}{d^2} \right]$$

2) Rotating frames

$$m\vec{a} = \vec{F} - 2m\vec{\Omega}\times\vec{v} - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$
$$= \vec{F} + \vec{F}_{CORIOLIS} + \vec{F}_{CENTRIFUGAL}$$

$$\vec{F}_{cent} = m\Omega^2\vec{\rho}$$

Near surface of earth $\quad \vec{g}_{eff} = \vec{g}_0 - \Omega^2 R_E \sin\theta\,\hat{\rho}$

$$\Rightarrow \quad m\vec{a} = m\vec{g}_{eff} + \vec{F}' - 2m\vec{\Omega}\times\vec{v}$$

↳ other non-gravitational forces

CORIOLIS EFFECTS:

- air circulation around low pressure areas (hurricanes)
- projectile motion
- Foucault pendulum