1)(a) The probability distribution is $F(x) = Ce^{-\alpha x}$.
First NORMALIZE. Note that the <u>distance</u> only takes on positive values

$$\int_0^\infty F(x)\,dx = 1 = \int_0^\infty Ce^{-\alpha x}\,dx \qquad y = \alpha x \qquad dx = \frac{dy}{\alpha}$$

$$1 = C\int_0^\infty e^{-y}\,\frac{dy}{\alpha} = \frac{C}{\alpha}(1) = 1 \qquad \text{so } \boxed{C = \alpha}$$

$$\lambda = \langle x \rangle = \int_0^\infty x\,Ce^{-\alpha x}\,dx = \alpha\int_0^\infty \left(\frac{y}{\alpha}\right)e^{-y}\left(\frac{dy}{\alpha}\right)$$

$$= \frac{1}{\alpha}\int_0^\infty ye^{-y}\,dy = \frac{1}{\alpha} \qquad \boxed{\lambda = 1/\alpha}$$

(b) $$P(x > 2\lambda) = \int_{2\lambda}^\infty F(x)\,dx = C\int_{2\lambda}^\infty e^{-\alpha x}\,dx$$

$$= \alpha\left[-\frac{1}{\alpha}e^{-\alpha x}\right]\Big/_{2\lambda}^{\infty} = \alpha\cdot\frac{1}{\alpha}e^{-2\lambda\alpha} = \boxed{e^{-2}}$$

2) The assumptions are i) circular orbits with $F = m\frac{v^2}{r}$
ii) $L = mvr = n\hbar$

$$F_0 = \frac{mv^2}{r} \quad * \qquad v = \frac{n\hbar}{mr}$$

$$F_0 = \frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = \frac{n^2\hbar^2}{mr^3} \qquad r^3 = \frac{n^2\hbar^2}{mF_0}$$

$$\boxed{r_n = \left[\frac{n^2\hbar^2}{mF_0}\right]^{\frac{1}{3}}}$$

For the energy

$$E = \frac{1}{2}mv^2 + V = \frac{1}{2}mv^2 + F_0 r$$

but from above

$$mv^2 = F_0 r \quad \Rightarrow \quad E = \frac{3}{2}F_0 r$$

$$\boxed{E_n = \frac{3}{2} F_0 \left[\frac{n^2\hbar^2}{mF_0}\right]^{\frac{1}{3}} = \frac{3}{2}\left[n^2\hbar^2 F_0^2/m\right]^{\frac{1}{3}}}$$

(b) For the state to be bound we need $r_n < a$

$$\left[\frac{n^2\hbar^2}{mF_0}\right] < a^3 \qquad n^2 < mF_0 a^3/\hbar^2$$

$$n < \left[mc^2 F_0 a^3/(\hbar c)^2\right]^{\frac{1}{2}}$$

$$= \left[(5.11\times10^5 eV)\left(\frac{1eV}{nm}\right)(8nm)^3 / (197.33 eV\cdot nm)^2\right]^{\frac{1}{2}}$$

$n < 10.25 \quad \Rightarrow \quad n=10$ is the last one

$\boxed{10 \text{ states}}$

3) $\psi = Ce^{-ax^2} \qquad V(x) = \frac{1}{2}kx^2$

$$\frac{d\psi}{dx} = -2ax\,Ce^{-ax^2} \;; \qquad \frac{d^2\psi}{dx^2} = -2aCe^{-ax^2} - 2ax(-2ax)e^{-ax^2}\cdot C$$

$$= \left[-2a + 4a^2x^2\right]Ce^{-ax^2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi$$

$$= -\frac{\hbar^2}{2m}\left[-2a + 4a^2x^2\right]Ce^{-ax^2} + \frac{1}{2}kx^2Ce^{-ax^2}$$

$$= \left[\frac{\hbar^2 a}{m} - 2\frac{\hbar^2a^2}{m}x^2 + \frac{1}{2}kx^2\right]Ce^{-ax^2}$$

We need this to be [constant]$\cdot\psi$ $\Rightarrow$ we need.

$$2\frac{\hbar^2a^2}{m} = \frac{1}{2}k \qquad a^2 = \frac{1}{4}km/\hbar^2 \qquad a = \frac{\sqrt{km}}{2\hbar}$$

and then

$$E = \frac{\hbar^2 a}{m} = \frac{\hbar^2}{m}\frac{\sqrt{km}}{2\hbar} \qquad \boxed{E = \frac{1}{2}\hbar\sqrt{\frac{k}{m}}}$$

4) According to Planck's hypothesis the only allowed energies for a mode of frequency ν are

$$E = 0, h\nu, 2h\nu, 3h\nu \cdots, nh\nu, \cdots$$

The probability of having energy E_n is given by the Boltzmann formula

$$P(E_n) = Ce^{-E_n/kT} = Ce^{-nh\nu/kT}$$

Here

$$h\nu = hc/\lambda = (1240\,eV \cdot nm)/600\,nm = 2.07\,eV$$

$$kT = (8.617 \times 10^{-5}\,eV/K)(1500\,K) = 0.129\,eV$$

so

$$h\nu/kT = 16 \qquad e^{-h\nu/kT} = e^{-16} = 1.14 \times 10^{-7}$$

$h\nu \gg kT$ so nearly all modes have $E = 0$

$$P(E = h\nu)/P(E = 0) = e^{-h\nu/kT} = 1.14 \times 10^{-7}$$

$$P(E = 0) \simeq 1 \qquad \Rightarrow \qquad \boxed{P(E = h\nu) \simeq 1.14 \times 10^{-7}}$$