Physics 448 **HOMEWORK SET 1** **Fall 2008**

Due Wednesday September 10

1) In class we showed that the Maxwell-Boltzmann velocity distribution has the form

$$f(v_x) = Ce^{-\lambda v_x^2/2}.$$

Determine the constants C and λ by requiring that $f(v_x)$ be normalized and that $\frac{1}{2}m\langle v_x^2\rangle = \frac{1}{2}kT$.

2) Calculate the probability that a helium atom in the upper atmosphere has a velocity v_z that exceeds the escape velocity (11.2 km/s). Assume a temperature of 1000 K (the gas in the far upper atmosphere is heated by solar wind). To evaluate the probability integral you may need to use some sort of approximation. Comment on the likelyhood that the earth's atmosphere has much helium.

3) A container of gas in thermal equilibrium at temperature T has a tiny pinhole that allows a small number of atoms to escape. In time Δt, atoms with higher v_x have a higher probability of escaping, so the v_x distribution of the escaping atoms goes like $h(v_x) \propto v_x f(v_x)$. The v_y and v_z distributions are the same inside and outside the box since the escape probability does not depend on those variables. Using this information find (a) the most probable value of v_x; (b) the mean value of v_x; and (c) the average kinetic energy of the atoms that escape.

4) To understand blackbody radiation we postulated the existence of electromagnetic standing waves in a cubic box extending from 0 to a in each rectangular dimension. Show that the electromagnetic standing wave

$$\mathcal{E}_x = A\cos\frac{l\pi}{a}x\,\sin\frac{m\pi}{a}y\,\sin\frac{n\pi}{a}z\,\cos\omega t \qquad \mathcal{B}_x = D\sin\frac{l\pi}{a}x\,\cos\frac{m\pi}{a}y\,\cos\frac{n\pi}{a}z\,\sin\omega t$$

$$\mathcal{E}_y = B\sin\frac{l\pi}{a}x\,\cos\frac{m\pi}{a}y\,\sin\frac{n\pi}{a}z\,\cos\omega t \qquad \mathcal{B}_y = E\cos\frac{l\pi}{a}x\,\sin\frac{m\pi}{a}y\,\cos\frac{n\pi}{a}z\,\sin\omega t$$

$$\mathcal{E}_z = C\sin\frac{l\pi}{a}x\,\sin\frac{m\pi}{a}y\,\cos\frac{n\pi}{a}z\,\cos\omega t \qquad \mathcal{B}_z = F\cos\frac{l\pi}{a}x\,\cos\frac{m\pi}{a}y\,\sin\frac{n\pi}{a}z\,\sin\omega t$$

satisfies Maxwell's Equations in free space for the appropriate choice of the constants A . . . F. [Hints: First use $\vec{\nabla}\cdot\vec{\mathcal{E}} = 0$ to find a relationship between A, B and C. Then determine D, E and F from $\vec{\nabla}\times\vec{\mathcal{E}} = -\frac{\partial\vec{\mathcal{B}}}{\partial t}$. The final two equations should then be satisfied for the correct value of ω.]

5) (a) Starting from Planck's formula for the blackbody distribution,

$$I(\nu) = (\frac{2\pi}{c^2})\nu^3\frac{h}{e^{(h\nu/kT)}-1},$$

show that the peak of the distribution occurs at frequency $\nu_m = a(kT/h)$ where a is some dimensionless constant. [Hint: The equation you get is transcendental, but the problem can be worked without actually solving the equation.]

(b) Show that the transcendental equation obtained in part (a) is solved for a = 2.8214.