Physics 448 **HOMEWORK SET 10** **Fall 2008**

Due Wednesday December 3

53) The energy eigenfunctions for the three-dimensional harmonic oscillator can be written in the form $\psi_{k,l,m}(x,y,z)$. The quantum numbers k, l and m take on integer values, 0,1,2, . . . , and the energy of the state is given by $E = (n+\frac{3}{2})\,\hbar\omega$ where $n = k+l+m$. Find the number of degenerate states for each energy, E_n.

54) (a) Use the rule $[p,x] = \frac{\hbar}{i}$ and the identity from part (c) of Problem 51 to show that $[p,x^n] = n(\frac{\hbar}{i})x^{n-1}$. Use proof by induction to find the result for arbitrary n. Work the problem using only general commutation relations – *i.e.* without ever writing $p = \frac{\hbar}{i}\frac{d}{dx}$.

(b) Show similarly that $[p^n,x] = n(\frac{\hbar}{i})p^{n-1}$.

55) (a) Use the formula

$$\frac{d}{dt}\langle q\rangle \;=\; \frac{1}{i\hbar}\langle\,[\mathrm{Q},\mathrm{H}]\,\rangle + \langle\frac{\partial \mathrm{Q}}{\partial t}\rangle$$

to find expressions for $\frac{d}{dt}\langle x\rangle$ in terms of $\langle p\rangle$ for the harmonic oscillator problem, $\mathrm{H} = \frac{p^2}{2m} + \frac{1}{2}kx^2$. The results from Problem 54 will be useful.

(b) In the same way, find an expression for $\frac{d}{dt}\langle p\rangle$ in terms of $\langle x\rangle$ for the harmonic oscillator.

(c) Use the results from (a) and (b) to derive a formula that relates $\frac{d^2}{dt^2}\langle x\rangle$ and $\langle x\rangle$. Start by taking the time derivative of the formula for $\frac{d}{dt}\langle x\rangle$ that you found in (a), and then use your result from (b) to eliminate p. Your result will be in the form of a differential equation for $\langle x\rangle$. Solve the differential equation to obtain the general solution for $\langle x\rangle$ as a function of time.

56) A particle of mass m is moving in a potential well $V(x) = Cx^n$ where n is an integer.

(a) Use your result from Problem 54 to work out the commutator $[px,H]$. You should get two terms, one proportional to T and one proportional to V.

(b) Use the result to prove the Virial Theorem, which states that for $V(x) = Cx^n$, the expectation values of V and T are related by $\langle T\rangle = \frac{n}{2}\langle V\rangle$ for **all energy eigenstates**. The trick is to realize that all expectation values are time independent for energy eigenstates, and so $\frac{d}{dt}\langle px\rangle = 0$.

(c) What does the Virial Theorem say about $\langle T\rangle$ and $\langle V\rangle$ for the harmonic oscillator problem?

(d) What does the Virial Theorem say about $\langle T\rangle$ and $\langle V\rangle$ for the hydrogen atom? (The theorem still holds if x is replaced by r.)

57) The quantities Y_l^m (or $|l,m\rangle$ in the notation of the text) are normalized simultaneous eigenfunctions of L^2 and L_z. If we stick to $l=1$, there are only three basis functions, $\psi_1 = Y_1^1$, $\psi_2 = Y_1^0$ and $\psi_3 = Y_1^{-1}$.

(a) Using these basis states write the operators L_x, L_y and L_z in 3×3 matrix form. HINTS: Remember that the $Q_{ij} = \langle\psi_i|Q|\psi_j\rangle$. Look at equations (7-20), (7-23) and (7-24), and remember that the eigenfunctions are orthogonal (since they have different L_z eigenvalues). Finally, notice that L_x and L_y are linear combinations of L_+ and L_-

(b) Show that your matrix operators satisfy the standard angular momentum commutation relations given in equations (7-6).