Physics 448 **HOMEWORK SET 11** **Fall 2008**

Due Friday December 12

58) This is a problem to work out and/or verify formulas for the components of $\vec{L}$ in spherical coordinates. The operators are already known in rectangular coordinates, $L_x = \frac{\hbar}{i}(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y})$, etc. To convert from rectangular to spherical coordinates use the chain rule; for example,

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial x}.$$

(a) Work out the formula for L_z starting from the expressions $r = \sqrt{x^2+y^2+z^2}$, $\phi = \tan^{-1}(y/x)$ and $\theta = \tan^{-1}(z/\sqrt{x^2+y^2})$. The final correct answer is

$$L_z = \frac{\hbar}{i}\frac{\partial}{\partial \phi}.$$

(b) The correct expressions for L_x and L_y in spherical coordinates are as follows:

$$L_x = \frac{\hbar}{i}\left[-\sin\phi\frac{\partial}{\partial\theta} - \cot\theta\,\cos\phi\frac{\partial}{\partial\phi}\right],$$

$$L_y = \frac{\hbar}{i}\left[\cos\phi\frac{\partial}{\partial\theta} - \cot\theta\,\sin\phi\frac{\partial}{\partial\phi}\right].$$

Starting from $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, and $z = r\cos\theta$, convert the L_x formula back to rectangular coordinates and show that you get the expected result. You do not need to do anything for L_y.

59) Starting from the results of problem 58 show that the operator $L^2 = L_x^2 + L_y^2 + L_z^2$ is just

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right].$$

60) The $\ell = 1$ spherical harmonics can easily be written in rectangular coordinates as well as in spherical coordinates. The results are

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\,\frac{x+iy}{r} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\,\frac{z}{r} \qquad Y_1^{-1} = \sqrt{\frac{3}{8\pi}}\,\frac{x-iy}{r}$$

(a) By making a cyclic permutation of the indicees, predict the form of three functions which you think would be simultaneous eigenfunctions of L_x and L^2. Check whether your prediction is correct by applying the L_x operator, in rectangular coordinate form, to each function.

(b) An electron in hydrogen is known to have $L^2 = 2\hbar^2$ (meaning $\ell = 1$) and $L_z = \hbar$. Suppose we measure L_x of that electron. List the possible outcomes of this measurement and find the probability of each possible outcome.

61) Find the expectation value of r for the $2s$ and $2p$ states in hydrogen.

62) (a) Find the expectation values of the potential energy for the ground state of hydrogen. Express the results both numerically (in electron volts) and in terms of a formula.

(b) Find the expectation value of the kinetic energy for the ground state using the fact that $\langle E\rangle = \langle T\rangle + \langle V\rangle$ and that $\langle E\rangle = E_n$ (the energy eigenvalue).